## Final Exam

Subject : Modern Robotics, Lecturer : Prof. Youngjin Choi, Date : June 18, 2020 (Contact e-mail : cyj@hanyang.ac.kr)
Notice that the answers should be written only in English, otherwise you will get a zero point.

Problem 1 ( 30 pt ) Assume a five-bar linkage is in its zero position. Let $\left(p_{x}, p_{y}\right)$ be the position of the $\{\mathrm{b}\}$-frame origin expressed in $\{\mathrm{s}\}$-frame coordinates, and let $\phi$ be the orientation of the $\{\mathrm{b}\}$ frame. Find the forward kinematics Jacobian $J_{a}$ from $\mathcal{V}_{s}=J_{a} \dot{q}_{a}$ when $B, D$ are actuated, i.e, $\dot{q}_{a}=\left(\dot{\theta}_{2}, \dot{\psi}_{1}\right)$ and $\dot{q}_{p}=\left(\dot{\theta}_{1}, \dot{\theta}_{3}, \dot{\psi}_{2}\right)$. You may make use of the matrix inversion of $\left[\begin{array}{ccc}1 & 1 & -1 \\ 0 & 1 & -3 \\ -3 & -2 & 2\end{array}\right]^{-1}=\left[\begin{array}{ccc}-2 & 0 & -1 \\ 4.5 & -0.5 & 1.5 \\ 1.5 & -0.5 & 0.5\end{array}\right]$.


Problem 2 (40pt) The figure illustrates an RP robot moving in a vertical plane. The mass of link 1 is $\mathrm{m}_{1}=1[\mathrm{~kg}]$ and the center of mass is a distance $L_{1}=1[m]$ from joint 1 . The scalar inertia of link 1 about an axis through the center of mass and out of the plane is $\mathcal{I}_{1}=1\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]$. The mass of link 2 is $\mathrm{m}_{2}=1[\mathrm{~kg}]$, the center of mass is a distance $\theta_{2}$ from joint 1 , and the scalar inertia of link 2 about its center of mass is $\mathcal{I}_{2}=1\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]$. Gravity $g$ acts downward on the page. Derive the equation of motion?

$$
M(\theta) \ddot{\theta}+c(\theta, \dot{\theta})+g(\theta)=\tau
$$



Problem 3 ( 30 pt ) For given a one-dof mass-spring-damper system of the form $\mathrm{m} \ddot{x}+b \dot{x}+k x=f$, where $f$ is the control force and $\mathrm{m}=4[\mathrm{~kg}]$ and $b=2[\mathrm{Ns} / \mathrm{m}]$ and $k=0.1[\mathrm{~N} / \mathrm{m}]$, determine the gains $k_{p}$ and $k_{d}$ so that the following PD controller

$$
f=k_{d}\left(\dot{x}_{d}-\dot{x}\right)+k_{p}\left(x_{d}-x\right)
$$

can yield the critical damping and the $2 \%$ settling time of $0.01[s]$, where $x_{d}=1$ and $\dot{x}_{d}=0$.

