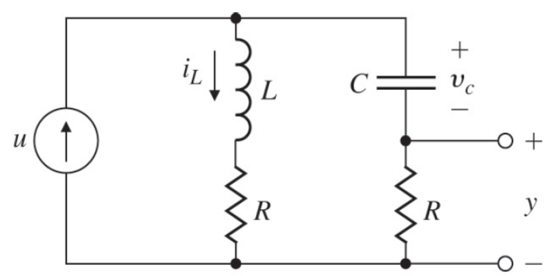
Final Exam

Subject : Control System Engineering 2, Lecturer : Prof. Youngjin Choi,
Date : Dec. 15, 2020 (Contact e-mail : cyj@hanyang.ac.kr)

Problem 1 (20pt) Consider the electric circuit shown in the figure.

(1.1) Write the state equations for the circuit, where the input $u(t)$ is a current, and the output $y(t)$ is a voltage. Let $x_1(t) = i_L(t)$ and $x_2(t) = v_c(t)$.

(1.2) What condition(s) on R, L, and C will guarantee that the system is controllable

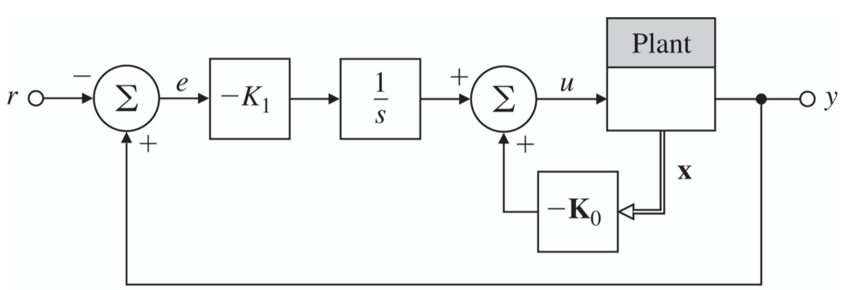


Problem 2 (25pt) Consider a system with state equation

$$\dot{x} = Ax + Bu \qquad\qquad y = Cx$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$



The system steady-state error performance can be made robust by augmenting the system with an integrator and using unity feedback; that is, by setting $\dot{x}_I = y - r$, where x_I is the state of the integrator. To see this, find state feedback $K_0 = [K_{01}, K_{02}]$ and K_1 of the form $u = -K_0 x - K_1 x_I$ so that the poles of the augmented system are at -3 ; $-2 \pm j3$.

Problem 3 (25pt) Consider the following compensator

$$D_c(s) = \frac{5}{s+5}$$

(3.1) Determine the sampling time T from $\omega_s = 25 \times \omega_{bw}$, where ω_s implies sampling rate and ω_{bw} means a bandwidth.

(3.2) Find the approximate model using Tustin's method ?

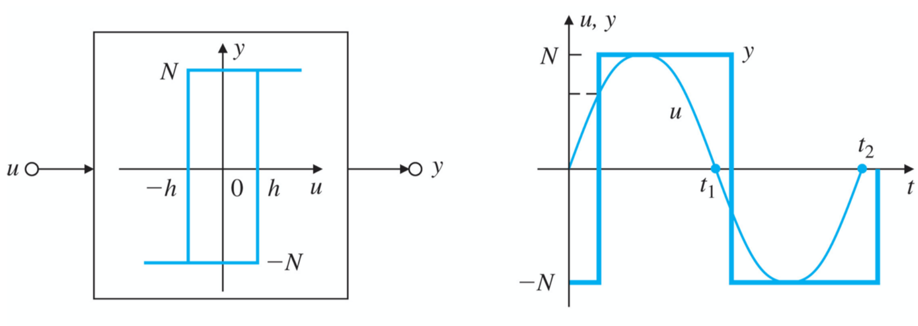
(3.3) Find the approximate model using ZOH ?

(3.4) Find the approximate model using MPZ ?

(3.5) Find the approximate model using MMPZ (modified MPZ) ?

Problem 4 (30pt) Consider the *relay function with hysteresis* shown in the below figure.

(4.1) Find the describing function (equivalent gain) for this nonlinearity when $u = a \sin \omega t$, where the output is a square wave with amplitude N as long as the input amplitude a is greater than the hysteresis level h.



(a) (b)

(4.2) Find the amplitude and the frequency of the limit cycle? where $N = 1$ and $h = 0.1$