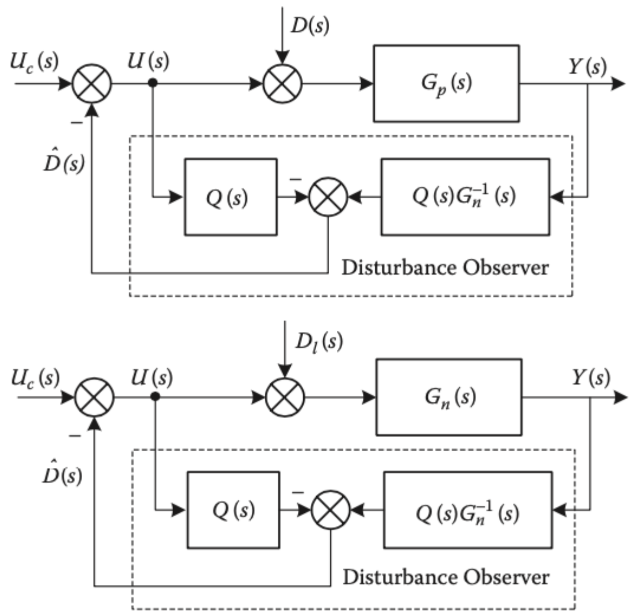
(DOB) 2. Linear Disturbance Estimator

1. Four commonly used *linear disturbance estimation* techniques are introduced.
2. The *frequency domain DOB*, which was proposed in the industrial application society in late 1980s, is presented first for both *minimum phase* and *nonminimum phase* cases.
3. The *time domain formulation of the DOB* is provided with a detailed analysis.
4. Finally, the *extended state observer* technique, which can simultaneously estimate states and disturbances, is presented.

(DOB) 2.2 Frequency Domain DOB (Minimum-Phase Case)



(a)
(b)

1. Consider a SISO linear minimum phase system

$$Y(s) = G_p(s)[U(s) + D(s)] \qquad \rightleftarrows \qquad Y(s) = G_n(s)[U(s) + D_l(s)]$$

2. Note that the DOB can estimate not only the *external disturbances* but also the *internal disturbances caused by model uncertainties*. To show how the DOB estimates the *lumped disturbances* (or extended disturbance in the PID control part) consisting of both the external and internal ones,

$$D_l(s) = G_n^{-1}(s)G_p(s)D(s) + [G_n^{-1}(s)G_p(s) - 1]U(s)$$
$$\hat{D}(s) = Q(s)G_n^{-1}(s)Y(s) - Q(s)U(s) = Q(s)D_l(s)$$

3. The *lumped disturbance estimation error* becomes

$$E_d(s) = \hat{D}(s) - D_l(s) = [Q(s) - 1]D_l(s)$$

   and it will tend to zero as time goes to infinity if the filter $Q(s)$ is selected as a *lowpass filter* form, that is, $\lim_{s\to 0} Q(s) = 1$.

4. It is also derived that the *output* can be represented as

$$\begin{aligned}
Y(s) &= G_p(s)[U(s) + D(s)] \\
&= \frac{G_p(s)}{1 - Q(s)}[U_c(s) - Q(s)G_n^{-1}(s)Y(s)] + G_p(s)D(s) \\
G_n(s)Y(s) &= \frac{G_p(s)}{1 - Q(s)}[G_n(s)U_c(s) - Q(s)Y(s)] + G_n(s)G_p(s)D(s) \\
(1 - Q(s))G_n(s)Y(s) &= G_p(s)G_n(s)U_c(s) - G_p(s)Q(s)Y(s) + (1 - Q(s))G_n(s)G_p(s)D(s) \\
\therefore \quad Y(s) &= \frac{G_p(s)G_n(s)}{(1 - Q(s))G_n(s) + G_p(s)Q(s)}U_c(s) + \frac{(1 - Q(s))G_n(s)G_p(s)}{(1 - Q(s))G_n(s) + G_p(s)Q(s)}D(s)
\end{aligned}$$

5. Clearly, if the filter is selected as a *lowpass* form, then $Q(s) = 1$ for low frequencies:

$$Y(s) \approx G_n(s)U_c(s) + 0 \cdot D(s)$$

   Above equation implies that the system with the frequency-domain DOB behaves as if it were the *nominal plant in the low-frequency* domain. It can be concluded that the *low-frequency-domain disturbances have been eliminated* from the system by feedforward compensation.

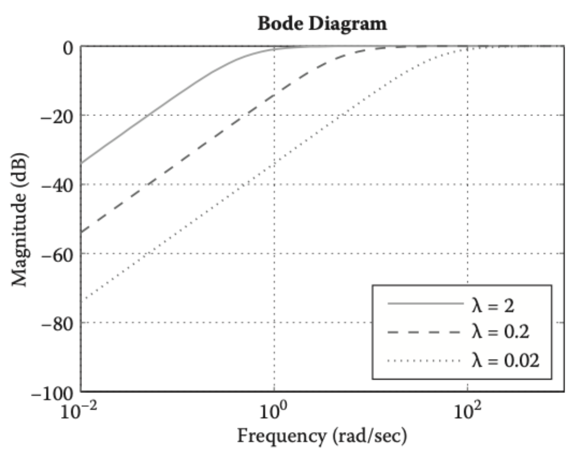
6. The performance of disturbance estimation is determined by the design of lowpass filter $Q(s)$.

   - The *relative degree* of $Q(s)$, that is, the order difference between the denominator and the numerator, should be *no less than that of the nominal model* $G_n(s)$. This design principle is to make sure that the control structure is *realizable*, i.e., $Q(s)G_n^{-1}(s)$ should be *proper*;
   - In the domain of low-frequency, $Q(s)$ approaches to 1, guaranteeing that the estimate of lumped disturbance approximately equals to the lumped disturbance.

7. Numerical example

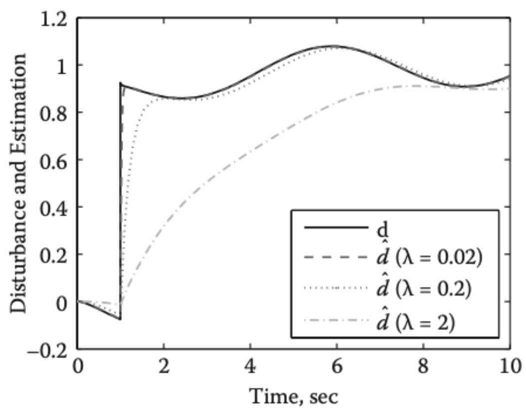
$$G_p(s) = \frac{s+3}{(s+1)(s+4)} \qquad G_n(s) = \frac{s+1}{(s+0.5)(s+2)} \qquad Q(s) = \frac{1}{\lambda s + 1}$$

   - In this case, the disturbance estimation accuracy depends on the selection of the filter parameter λ in $Q(s)$. Actually, the *property of disturbance estimation* is determined by the frequency characteristics of transfer function $1 - Q(s)$.

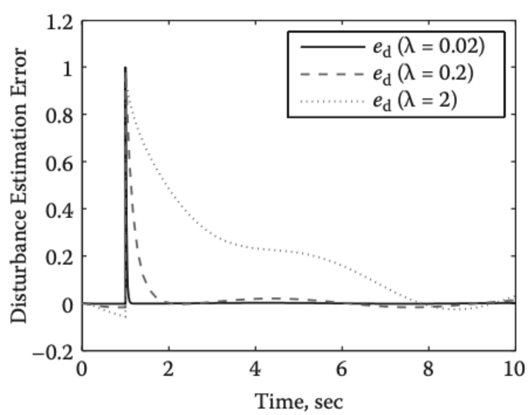


- The smaller the filter parameter λ is, the smaller the magnitude of transfer function $1 - Q(s)$ is.
- Let us apply the external disturbance as follows. A smaller filter parameter λ has brought a better transient dynamics in estimation and a smaller static disturbance estimation error, while a lager filter parameter has resulted in a larger disturbance estimation error.

$$d(t) = \begin{cases} \sin t & 0 \le t \le 1 \\ 1 + \sin t & t > 1 \end{cases}$$



(a)



(b)

(DOB) 2.2 Frequency Domain DOB (Nonminimum-Phase Case)

1. Consider the following nominal transfer function

$$G_n(s) = \frac{k_p(-\beta s + 1)}{(\tau_{p1}s + 1)(\tau_{p2}s + 1)} e^{-\tau s}$$

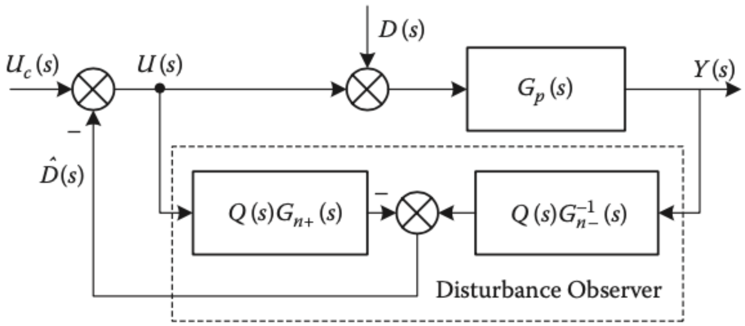
2. Suppose that the filter in the disturbance observer is chosen as a first-order low-pass form

$$Q(s) = \frac{1}{\lambda s + 1}$$

3. If we use the previous method for the minimum phase case to construct the disturbance observer for system, it yields

$$G_n^{-1}(s)Q(s) = \frac{(\tau_{p1}s + 1)(\tau_{p2}s + 1)}{k_p(-\beta s + 1)(\lambda s + 1)} e^{\tau s}$$

   Note that the zeros of the nominal model become the poles of DOB. This results in an *unstable observer* and thus the possibility of unbounded disturbance estimation.



4. To this end, The model has to be factored out before using the model inverse for the observer design. The most widely used method would be *all-pass factorization* $\frac{-\beta s+1}{\beta s+1}$, which places the RHP zero in the non-invertible part of the nominal model, and it also places a pole at the reflection of the RHP zero.

5. It is factored as follows:

$$\text{stable and minimum} \quad G_{n-}(s) = \frac{k_p(\beta s+1)}{(\tau_{p1}s+1)(\tau_{p2}s+1)} \qquad \text{all-pass} \quad G_{n+}(s) = \frac{-\beta s+1}{\beta s+1}e^{-\tau s}$$

6. From the above DOB configuration for *nonminimum phase plant*, we have the output

$$\begin{aligned}
Y(s) &= G_p(s)[U(s)+D(s)] \\
&= \frac{G_p(s)}{1-Q(s)G_{n+}(s)}[U_c(s) - Q(s)G_{n-}^{-1}(s)Y(s)] + G_p(s)D(s) \\
G_{n-}(s)Y(s) &= \frac{G_p(s)}{1-Q(s)G_{n+}(s)}[G_{n-}(s)U_c(s) - Q(s)Y(s)] + G_{n-}(s)G_p(s)D(s) \\
(1-Q(s)G_{n+}(s))G_{n-}(s)Y(s) &= G_p(s)G_{n-}(s)U_c(s) - G_p(s)Q(s)Y(s) + (1-Q(s)G_{n+}(s))G_{n-}(s)G_p(s)D(s) \\
\therefore \quad Y(s) &= \frac{G_p(s)G_{n-}(s)}{G_{n-}(s) - G_n(s)Q(s) + G_p(s)Q(s)}U_c(s) + \frac{G_{n-}(s)(1-G_{n+}(s)Q(s))G_p(s)}{G_{n-}(s) - G_n(s)Q(s) + G_p(s)Q(s)}D(s)
\end{aligned}$$

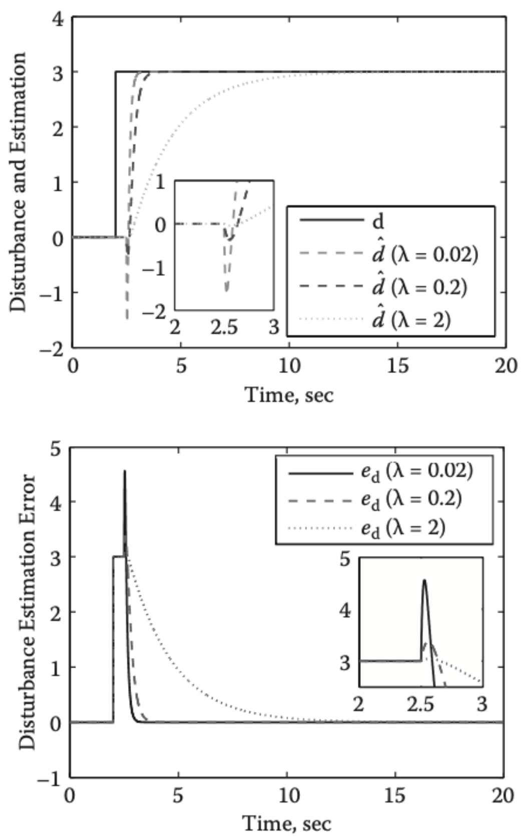
7. Note that the factorization ensures that $\lim_{s \to 0} G_{n+}(s) = 1$ and thus $\lim_{s \to 0} G_n(s) = G_{n-}(s)$, and also the filter $\lim_{s \to 0} Q(s) = 1$ is selected as a low-pass form, it follows for low frequencies

$$Y(s) \approx G_{n-}(s)U_c(s) + 0 \cdot D(s) \quad \text{for low frequencies}$$

where the system with frequency domain DOB behaves as if it were the nominal plant $G_{n-}(s)$ in the low frequency domain.

8. The low-frequency-domain disturbances for such nonminimum phase system have been eliminated from the system by feedforward compensation.

9. Numerical example, consider the step disturbance $d(t) = 3$ for $t \geq 2[s]$.

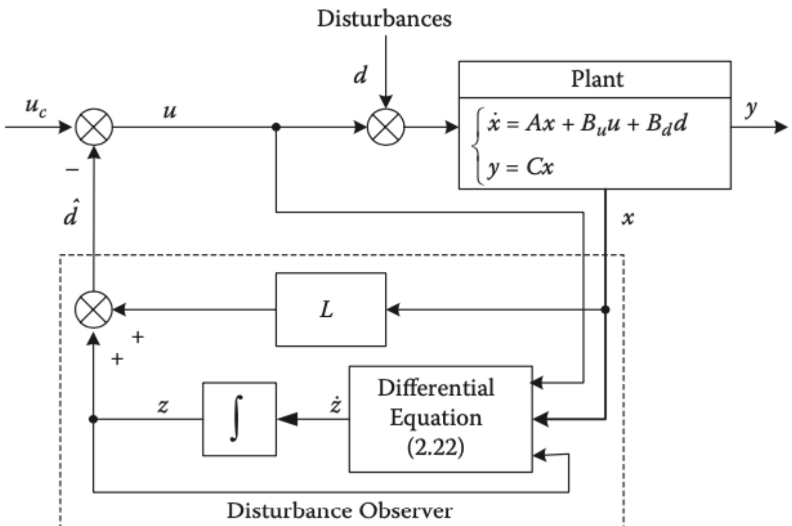


(a)

(b)

- The smaller the filter parameter is, the faster the convergence rate of estimation error is.
- However, a smaller filter parameter will result in a *larger nonminimum phase effects* of error dynamics of DOB.
- There exists a *trade-off* between faster disturbance-estimation dynamics and smaller nonminimum phase effects.

(DOB) 2.3 Time Domain DOB



1. Consider a MIMO linear system with disturbances, depicted by

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_d d \\ y &= Cx \end{aligned}$$

2. The following time-domain DOB can be employed to estimate the disturbances

$$\begin{aligned} \dot{z} &= -LB_d(z + Lx) - L(Ax + B_u u) \\ \hat{d} &= z + Lx \end{aligned}$$

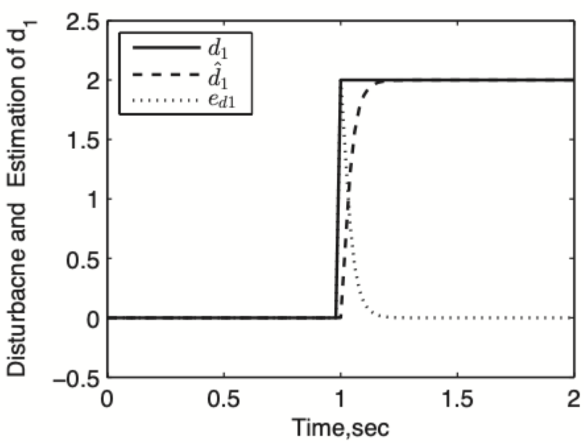
where $\hat{d}$ the disturbance estimation vector, z the internal variable vector of the observer, and L the observer gain matrix to be designed.

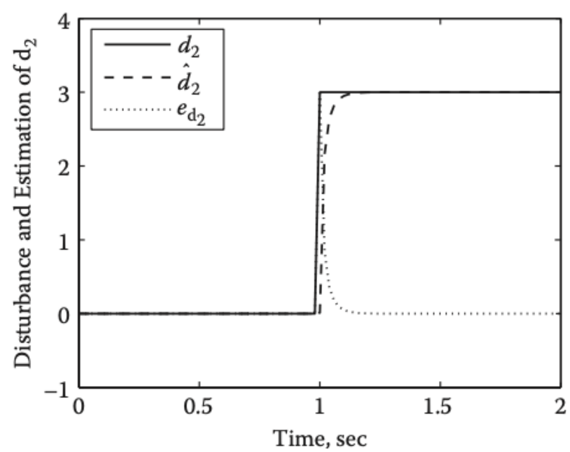
3. The disturbance estimation error and its time-derivative are obtained as

$$
\begin{aligned}
e_d &= \hat{d} - d \\
\dot{e}_d &= \dot{\hat{d}} - \dot{d} = \dot{z} + L\dot{x} - \dot{d} \\
&= [-LB_d\hat{d} - L(Ax + B_u u)] + L[Ax + B_u u + B_d d] - \dot{d} = -LB_d[\hat{d} - d] - \dot{d} \\
&= -LB_d e_d - \dot{d}
\end{aligned}
$$

where the estimation error system is *asymptotically stable* with appropriately chosen parameter L such that $-LB_d$ is *Hurwitz* when the disturbance is a *constant*.

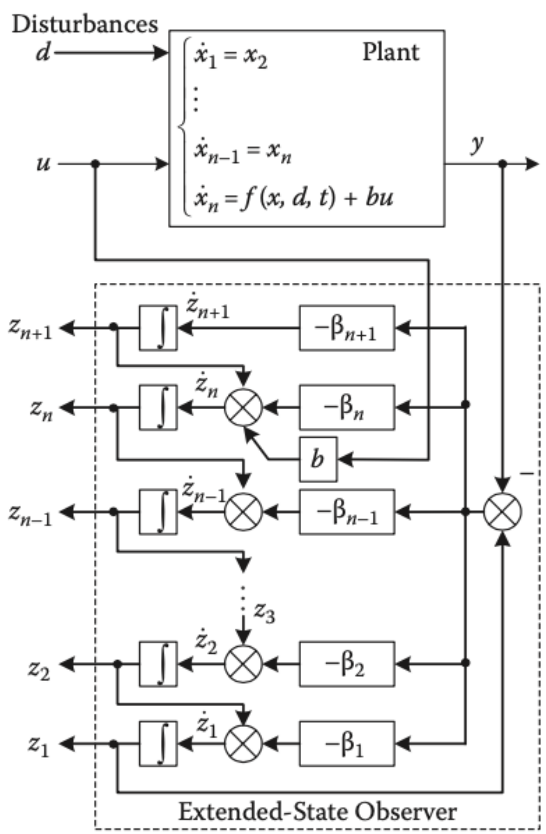
4. Numerical simulation,





The time-domain DOB here can be used for both minimum and nonminimum phase MIMO linear systems. However, it requires *all the state information* for observer design, while the frequency-domain DOB only uses the output and input information.

(DOB) 2.4 Extended State Observer (ESO)



1. Consider a class of SISO uncertain systems with order of n, described by the following differential equation

$$y^{(n)}(t) = f(y(t), \dot{y}(t), \ddot{y}(t), \cdots, y^{(n-1)}(t), d(t), t) + bu(t)$$

where $f(y(t), \dot{y}(t), \ddot{y}(t), \cdots, y^{(n-1)}(t), d(t), t)$ is the *lumped disturbances* consisting of the external one $d(t)$ and internal ones caused by model uncertainties.

2. Define new states using the *integrator chains* as follows:

$$\begin{array}{lllll} x_1 = y & x_2 = \dot{y} & \cdots & x_{n-1} = y^{(n-2)} & x_n = y^{(n-1)} \\ \dot{x}_1 = x_2 & \dot{x}_2 = x_3 & \cdots & \dot{x}_{n-1} = x_n & \dot{x}_n = f(x_1, x_2, \cdots, x_n, d, t) + bu \end{array}$$

3. The *augmented state variable* is introduced in the framework of an ESO to linearize system.

$$x_{n+1} = f(x_1, x_2, \cdots, x_n, d, t)$$

Now, the extended-state equation and its linear ESO are given by

$$\begin{array}{rcl} \dot{x}_1 = x_2 & \rightarrow & \dot{z}_1 = z_2 - \beta_1(z_1 - y) \\ \dot{x}_2 = x_3 & \rightarrow & \dot{z}_2 = z_3 - \beta_2(z_1 - y) \\ \vdots & & \vdots \\ \dot{x}_{n-1} = x_n & \rightarrow & \dot{z}_{n-1} = z_n - \beta_{n-1}(z_1 - y) \\ \dot{x}_n = x_{n+1} + bu & \rightarrow & \dot{z}_n = z_{n+1} - \beta_n(z_1 - y) + bu \\ \dot{x}_{n+1} = h(t) & \rightarrow & \dot{z}_{n+1} = -\beta_{n+1}(z_1 - y) \\ y = x_1 & & \end{array}$$

where $h(t) = \dot{f}(x_1, x_2, \cdots, x_n, d, t)$ is assumed to be *bounded*, and $z_1, z_2, \cdots, z_n, z_{n+1}$ are estimates of states $x_1, x_2, \cdots, x_n, x_{n+1}$, respectively, and $\beta_1, \beta_2, \cdots, \beta_{n+1}$ are the observer gains

4. The *estimation error* $e_i = z_i - x_i$ dynamics is obtained by

$$\dot{e}_1 = e_2 - \beta_1 e_1 \qquad \dot{e}_2 = e_3 - \beta_2 e_1 \qquad \cdots \qquad \dot{e}_n = e_{n+1} - \beta_n e_1 \qquad \dot{e}_{n+1} = -\beta_{n+1} e_1 - h(t)$$

5. Numerical example, consider the following second-order system

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = e^{x_1} + d + u \qquad y = x_1$$

where the *lumped disturbances* is taken as $f(x,d) = e^{x_1} + d$ and the *external disturbance* is taken as a constant one: $d(t) = 3$ for $t \geq 6[s]$.

- The ESO for system is designed as

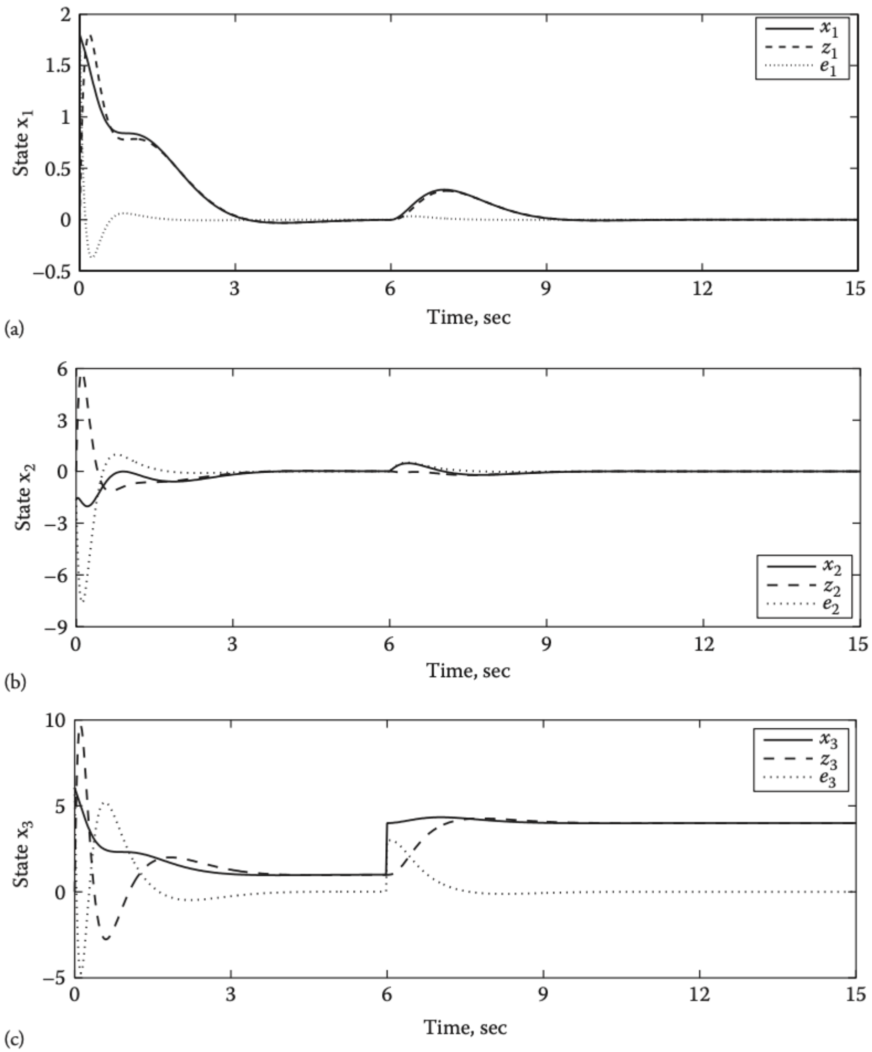
$$\dot{z}_1 = z_2 - \beta_1(z_1 - y) \qquad \dot{z}_2 = z_3 - \beta_2(z_1 - y) + u \qquad \dot{z}_3 = -\beta_3(z_1 - y)$$

  where the parameters are chosen as $\beta_1 = 15, \beta_2 = 75, \beta_3 = 125$.

- Here, a composite control law is designed as

$$u = -4z_1 - 4z_2 - z_3$$

- The designed ESO can effectively estimate both *states and disturbances* in the presence of plant uncertainties.
- Note that the ESO only demands for output and input information of the system. So it is usually employed for *output feedback based disturbance rejection*.


State x_1
Time, sec
x_1
z_1
e_1
(a)
State x_2
Time, sec
x_2
z_2
e_2
(b)
State x_3
Time, sec
x_3
z_3
e_3
(c)

(DOB) 3. Nonlinear Disturbance Observer (DOB)

1. Two kinds of practical nonlinear DOB are presented.

2. The first one is referred to as *constant nonlinear DOB*, the estimation error of which converges to zero if the disturbance is constant one.

3. The second kind is referred to as *harmonic nonlinear DOB*. The disturbances in this case are not limited to constant ones anymore.

4. The amplitudes of disturbances are not necessarily required for the observer design and analysis. However, in order to achieve accurate estimation, the *frequencies of harmonic disturbances* under consideration have to be known.

(DOB) 3.2 Nonlinear DOB for Constant Disturbances

1. Consider a class of *affine nonlinear systems*, depicted by

$$
\begin{aligned}
\dot{x} &= f(x) + g_1(x)u + g_2(x)d \\
y &= h(x)
\end{aligned}
$$

   where it is assumed that $f(x), g_1(x), g_2(x), h(x)$ are *smooth functions* in terms of x . The disturbances under consideration are supposed to be constant but unknown.

2. To estimate the unknown disturbances d , a basic nonlinear DOB is suggested as

$$
\dot{\hat{d}} = l(x)[\dot{x} - f(x) - g_1(x)u - g_2(x)\hat{d}]
$$

   where $\hat{d}$ denotes the disturbance estimation vector, and $l(x)$ is the nonlinear gain function.

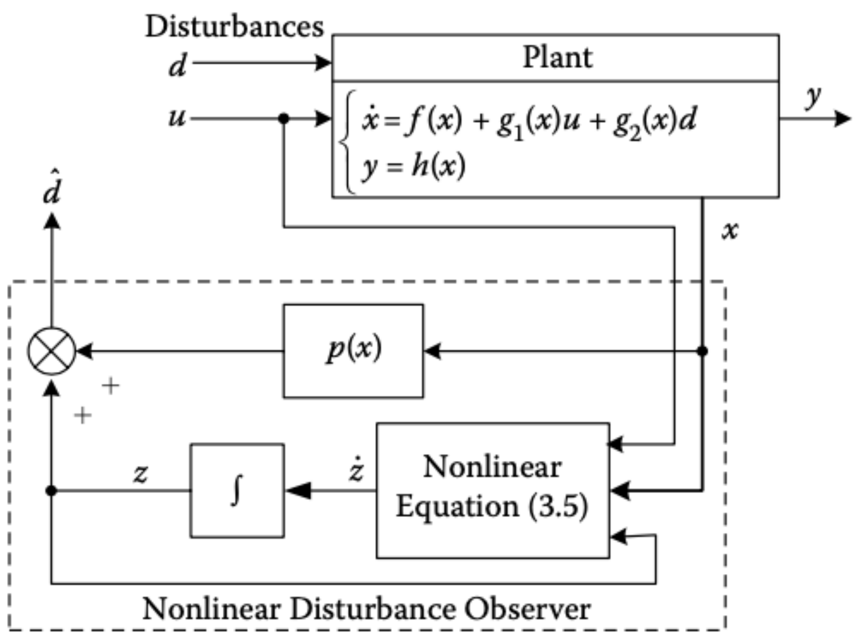
3. The *disturbance estimation error* and its time-derivative are obtained as

$$
\begin{aligned}
e_d &= \hat{d} - d \\
\dot{e}_d &= \dot{\hat{d}} - \dot{d} = l(x)[\dot{x} - f(x) - g_1(x)u - g_2(x)\hat{d}] - \dot{d} = -l(x)g_2(x)e_d - \dot{d}
\end{aligned}
$$

   $e_d(t) \to 0$ as $t \to \infty$ if gain $l(x)$ is chosen such that the system is asymptotically stable.

4. It should be pointed out that for implementation of the above disturbance observer, the *derivative of the state* is required, which may need an additional sensor for measuring it.

(DOB) 3.2.2 An Enhanced Formulation



1. An enhanced nonlinear DOB is introduced to estimate the constant disturbance

$$\dot{z} = -l(x)g_2(x)[z + p(x)] - l(x)[f(x) + g_1(x)u]$$
$$\hat{d} = z + p(x)$$

where z is the internal state of the nonlinear observer, and $p(x)$ is the nonlinear function to be designed. Let us determine the nonlinear disturbance observer gain $l(x)$ by

$$l(x) = \frac{\partial p(x)}{\partial x}$$

2. The *disturbance estimation error* is governed by

$$\begin{aligned}
\dot{e}_d &= \dot{\hat{d}} - \dot{d} \\
&= \dot{z} + \frac{\partial p(x)}{\partial x}\dot{x} - \dot{d} = \dot{z} + l(x)\dot{x} - \dot{d} \\
&= \Big\{-l(x)g_2(x)\hat{d} - l(x)[f(x) + g_1(x)u]\Big\} + l(x)[f(x) + g_1(x)u + g_2(x)d] - \dot{d} \\
&= -l(x)g_2(x)[\hat{d} - d] - \dot{d} \\
&= -l(x)g_2(x)e_d - \dot{d}
\end{aligned}$$

where the nonlinear DOB can estimate unknown constant disturbances if the observer gain $l(x)$ is chosen such that system is asymptotically stable.

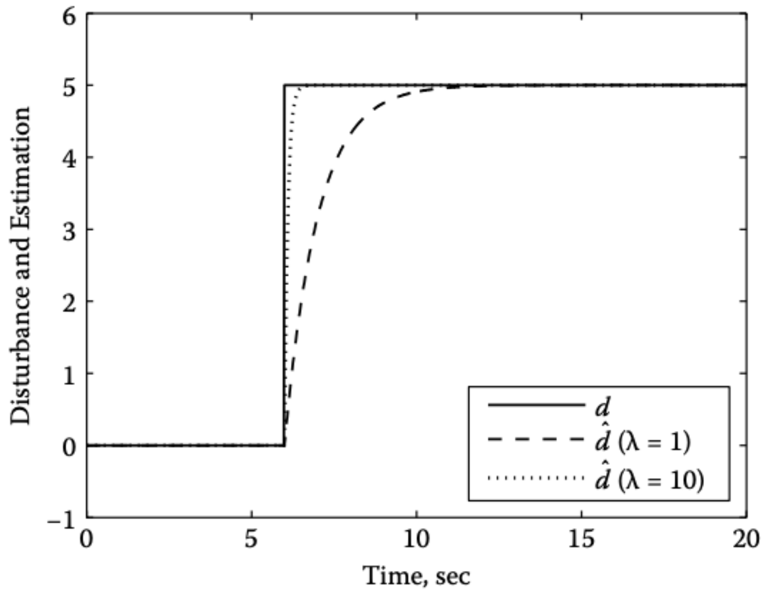
3. Numerical example, consider the following nonlinear system with disturbances

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 + \sin^2 x_1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d \qquad \rightarrow \qquad \dot{x} = f(x) + g_1(x)u + g_2(x)d$$

   - Suppose that a constant disturbance is imposed on system, i.e., $d(t) = 5$ for $t \geq 6[s]$. The nonlinear DOB is designed according to the above procedures, $p(x) = \lambda x_2$ and $l(x)g_2(x) = [0 \;\; \lambda][0 \;\; 1]^T = \lambda$, and the composite controller is designed as

$$u = \frac{-k_1 x_1 - k_2 x_2 - x_1 x_2 - \hat{d}}{1 + \sin^2 x_1}$$

     where $k_1 = 10$ and $k_2 = 30$.
   - It can be observed from the figure that the nonlinear DOB could estimate constant disturbance asymptotically.
   - The larger the observer parameter λ, the quicker the convergence rate of observer.


Disturbance and Estimation
Time, sec
6
5
4
3
2
1
0
−1
0
5
10
15
20
d
$\hat{d}$ (λ = 1)
$\hat{d}$ (λ = 10)

(DOB) 3.3 Nonlinear DOB for General Exogenous Disturbances

1. For given the nonlinear system, the disturbances are supposed to be harmonic ones with known frequency, but unknown amplitude and phase rather than constant ones.

2. It is supposed that the disturbances are generated by the following exogenous system

$$\dot{\xi} = A\xi \qquad\qquad d = C\xi$$

3. The following basic harmonic nonlinear disturbance observer can be employed to estimate the harmonic disturbances in system

$$\dot{\hat{\xi}} = A\hat{\xi} + l(x)[\dot{x} - f(x) - g_1(x)u - g_2(x)\hat{d}] \qquad\qquad \hat{d} = C\hat{\xi}$$

where $\hat{\xi}$ is the internal state variable of observer. For example, if $d(t) = \sin t$, then $\ddot{d} + d = 0$ with an initial conditions $d(0) = 1$ and $\dot{d}(0) = 0$, and thus we have

$$\frac{d}{dt}\begin{bmatrix} d \\ \dot{d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} d \\ \dot{d} \end{bmatrix} \qquad \rightarrow \qquad \dot{\xi} = A\xi$$

$$d = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} d \\ \dot{d} \end{bmatrix} \qquad \rightarrow \qquad d = C\xi$$

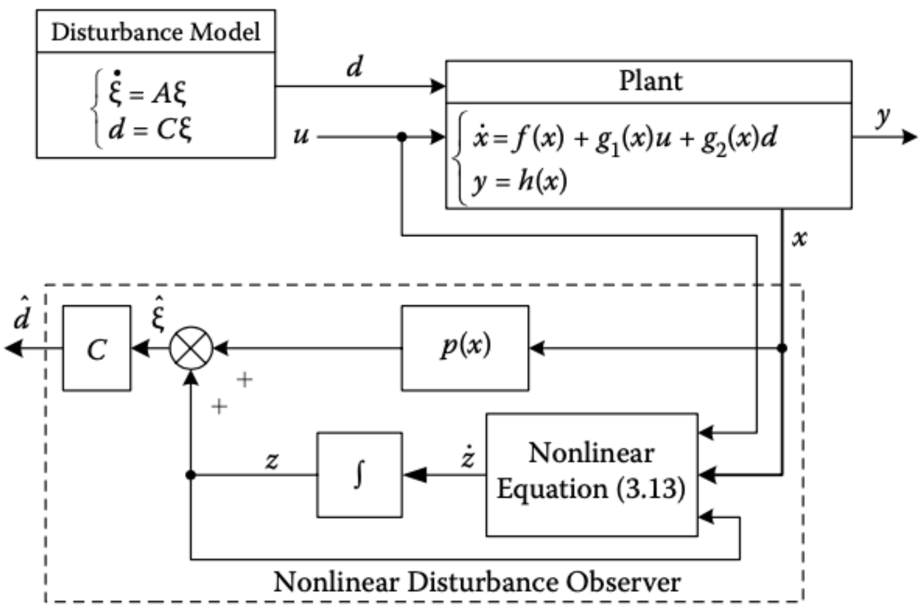
4. The *observer estimation error* and its time-derivative are obtained as

$$
\begin{aligned}
e_\xi &= \hat{\xi} - \xi \\
\dot{e}_\xi &= \dot{\hat{\xi}} - \dot{\xi} = \left\{ A\hat{\xi} + l(x)[\dot{x} - f(x) - g_1(x)u - g_2(x)\hat{d}] \right\} - A\xi \\
&= Ae_\xi + l(x)g_2(x)(d - \hat{d}) \\
&= Ae_\xi + l(x)g_2(x)(C\xi - C\hat{\xi}) \\
&= [A - l(x)g_2(x)C]e_\xi
\end{aligned}
$$

where $\hat{\xi}(t)$ approaches to $\xi(t)$ asymptotically if $l(x)$ is chosen such that $[A - l(x)g_2(x)C]$ is asymptotically stable regardless of x.

5. However, such a basic *harmonic nonlinear DOB* still meets the problem of implementation for practical application due to the requirement of the time-derivatives of the states.

(DOB) 3.3.2 An Enhanced Formulation for Harmonic nonlinear DOB



1. Based on the original version of harmonic nonlinear DOB, an enhanced version is depicted by

$$
\begin{aligned}
\dot{z} &= [A - l(x)g_2(x)C][z + p(x)] - l(x)[f(x) + g_1(x)u] \\
\hat{\xi} &= z + p(x) \\
\hat{d} &= C\hat{\xi}
\end{aligned}
$$

2. The *disturbance estimation error* is governed by

$$
\begin{aligned}
\dot{e}_\xi &= \dot{\hat{\xi}} - \dot{\xi} \\
&= \dot{z} + \frac{\partial p(x)}{\partial x}\dot{x} - \dot{\xi} = \dot{z} + l(x)\dot{x} - \dot{\xi} \\
&= \{[A - l(x)g_2(x)C][z + p(x)] - l(x)[f(x) + g_1(x)u]\} + l(x)[f(x) + g_1(x)u + g_2(x)d] - A\xi \\
&= [A - l(x)g_2(x)C]\hat{\xi} + l(x)g_2(x)C\xi - A\xi \\
&= [A - l(x)g_2(x)C](\hat{\xi} - \xi) \\
&= [A - l(x)g_2(x)C]e_\xi
\end{aligned}
$$

where $l(x)$ should be chosen so that $A - l(x)g_2(x)C$ is stable.

3. Suppose that the relative degree from the disturbance to the output, r, is uniformly well-defined. The nonlinear function $p(x)$ is designed as

$$
p(x) = K\frac{d^{(r-1)}h(x)}{dx^{(r-1)}}f(x) = KL_f^{(r-1)}h(x)
$$

It follows that

$$
l(x) = \frac{\partial p(x)}{\partial x} = K\frac{\partial L_f^{(r-1)}h(x)}{\partial x}
$$

4. Reconsider the *observation error dynamics*

$$
\dot{e}_\xi = [A - l(x)g_2(x)C]e_\xi = \left[A - K\frac{\partial L_f^{(r-1)}h(x)}{\partial x}g_2(x)C\right]e_\xi = \left[A - KL_{g_2}L_f^{(r-1)}h(x)C\right]e_\xi
$$

where $L_{g_2}L_f^{(r-1)}h(x)$ can be divided as

$$L_{g_2}L_f^{(r-1)}h(x) = \alpha_0 + \alpha_1(x)$$

in which $\alpha_0(x)$ belongs to class $\mathcal{K}$ function.

5. Check the *stability* using Lyapunov function:

$$\begin{aligned}
V(e_\xi) &= e_\xi^T P e_\xi \\
\dot{V} &= \dot{e}_\xi^T P e_\xi + e_\xi^T P \dot{e}_\xi \\
&= e_\xi^T [A - K\alpha_0 C - K\alpha_1(x)C]^T P e_\xi + e_\xi^T P[A - K\alpha_0 C - K\alpha_1(x)C]e_\xi \\
&= e_\xi^T [(A - K\alpha_0 C)^T P + P(A - K\alpha_0 C)]e_\xi - 2e_\xi^T PK\alpha_1(x)Ce_\xi
\end{aligned}$$

where we choose P satisfying

$$(A - K\alpha_0 C)^T P + P(A - K\alpha_0 C) = -Q \qquad PK = C^T$$

then we have

$$\begin{aligned}
\dot{V} &= -e_\xi^T Q e_\xi - 2e_\xi^T [C^T \alpha_1(x) C]e_\xi \\
&= -e_\xi^T Q e_\xi - 2(\hat{d} - d)^T \alpha_1(x)(\hat{d} - d) < 0
\end{aligned}$$

in which the estimation $\hat{d}$ yielded by harmonic nonlinear DOB converges to the disturbance d globally exponentially.

6. For simulation studies, please refer to the numerical data in the textbook.
   It can be observed from the figure that the nonlinear DOB could asymptotically estimate the general exogenous disturbance.