- 전형적인 **2**차 시스템의 감쇠비(ζ)와 위상여유(**PM**) 사이의 관계

  1. 단위 되먹임을 가진 개루프 **2**차 시스템

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

  2. 감쇠비 ζ와 위상여유 **PM** 사이의 관계:
  (1) 이득교차 주파수 $|G(j\omega_g)| = 1$

$$\frac{\omega_n^2}{|\omega_g|\sqrt{\omega_g^2 + 4\zeta^2\omega_n^2}} = 1 \qquad \omega_n^4 = \omega_g^4 + 4\zeta^2\omega_n^2\omega_g^2 \qquad \omega_g = \omega_n\sqrt{\sqrt{4\zeta^4 + 1} - 2\zeta^2}$$

  (2) 위상각

$$\angle G(j\omega_g) = -90^\circ - \tan^{-1}\frac{\omega_g}{2\zeta\omega_n}$$

  (3) 위상여유 **: [using** $90^\circ - \tan^{-1}\alpha = \tan^{-1}\frac{1}{\alpha}$**]**

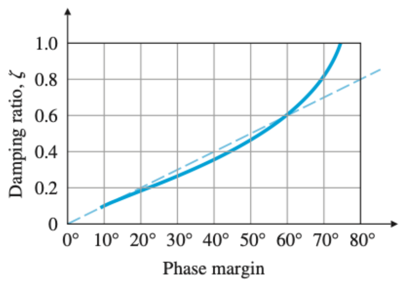
$$\begin{aligned} PM &= 180^\circ + \angle G(j\omega_g) \\ &= 90^\circ - \tan^{-1}\frac{\omega_g}{2\zeta\omega_n} \\ &= \tan^{-1}\frac{2\zeta}{\sqrt{\sqrt{4\zeta^4 + 1} - 2\zeta^2}} \end{aligned}$$

- 2차 시스템의 계단 입력에 대한 과도응답과 주파수응답 사이의 상호관계:
  - 감쇠비와 위상여유는 직접적인 관계를 가진다. $0 \leq \zeta \leq 0.6$에 대해서는 근사적으로 다음이 성립한다.

$$\zeta \approx \frac{PM}{100}$$

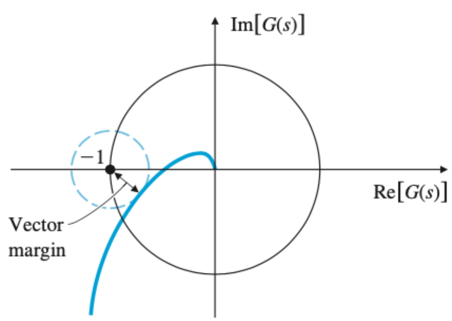
  - ω_r과 ω_d는 작은 ζ에 대해서는 거의 같다
  - ζ가 작을 때, M_r과 M_p 모두 커지긴 하지만, M_r은 대단히 커지고, M_p는 **1**을 **(100%**를**)** 초과하지 않음.

Figure 6.36
Damping ratio versus PM



- Vector margin is defined to be the distance to the $-1 + j0$ point from the closest approach of the Nyquist plot

Figure 6.38
Definition of the vector margin on the Nyquist plot

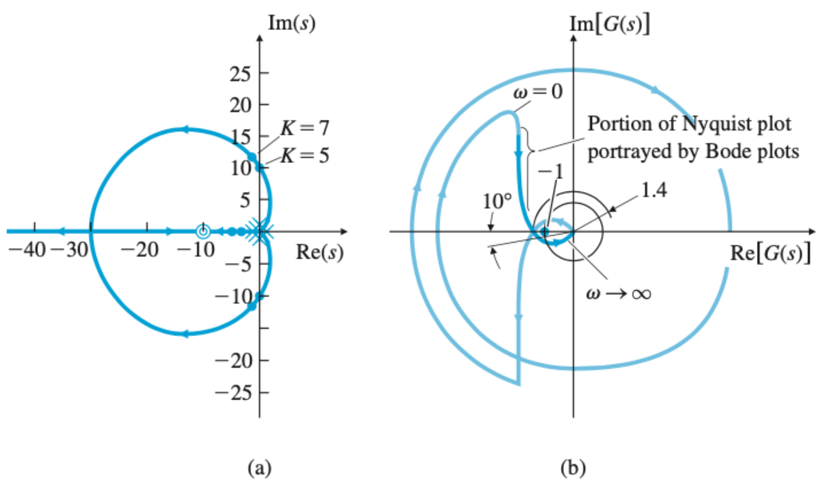


- (Example 6.12) Determine the stability property as a function of K

$$G(s) = \frac{K(s+10)^2}{s^3}$$
$$|G(j\omega)| = \frac{K(\omega^2+100)}{|\omega|^3}$$
$$\angle G(j\omega) = 2\tan^{-1}\frac{\omega}{10} - 270^\circ$$

  - $\omega = +0$ 일때, 크기 $= \infty$, 위상각 $= -270^\circ$
  - $\omega = +10$ 일때, 크기 $= \frac{K}{5}$, 위상각 $= -180^\circ$
  - $\omega = +\infty$ 일때, 크기 $= 0$, 위상각 $= -90^\circ$
  - As $s : -0 \rightarrow +0$ with a radius ϵ: $G(s)$는 양의 무한대 원으로 맵핑된다.
  - if $K < 5$, then $P = 0$, $N = 2$이므로, $Z = P + N = 2$가 되어 불안정하다.
  - if $K > 5$, then $P = 0$, $N = 0$이므로, $Z = P + N = 0$가 되어 안정하다.

Figure 6.40
System in which increasing gain leads from instability to stability: (a) root locus; (b) Nyquist plot



- (Example 6.13)

$$G(s)=\frac{85(s+1)(s^2+2s+43.25)}{s^2(s^2+2s+82)(s^2+2s+101)}$$

At $\omega_g = 0.75, 9.0$, and $10.1 rad/s$, PM's become 37도, 80도, 40도가 됨.

Figure 6.42
Bode plot of the system in Example 6.13

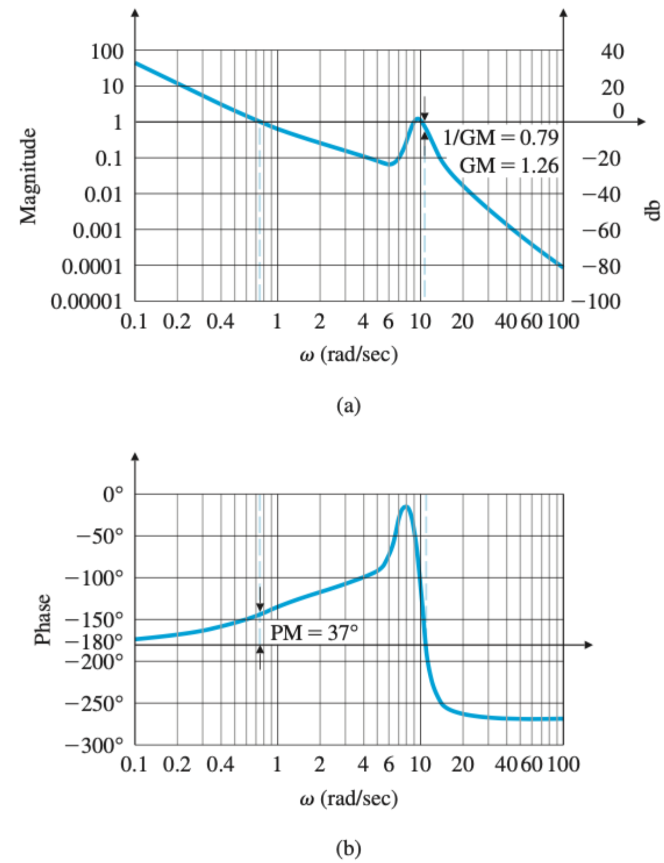
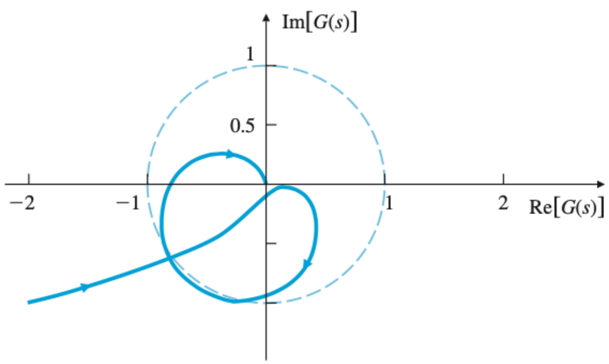


Figure 6.41
Nyquist plot of the complex system in Example 6.13



5 Bode's Gain-Phase Relationship

- For any stable minimum phase system, the phase of $G(j\omega)$ is uniquely related to the magnitude of $G(j\omega)$
- Adjust the slope of the magnitude curve $|KG(j\omega)|$ so that it crosses over magnitude 1 (=0dB) with a slope of -1 (=-20dB/decade) for a decade around gain crossover frequency ω_g
- (Example 6.14) Design K and T_D of PD controller satisfying $\omega_g = 0.2$ and $PM = 75°$

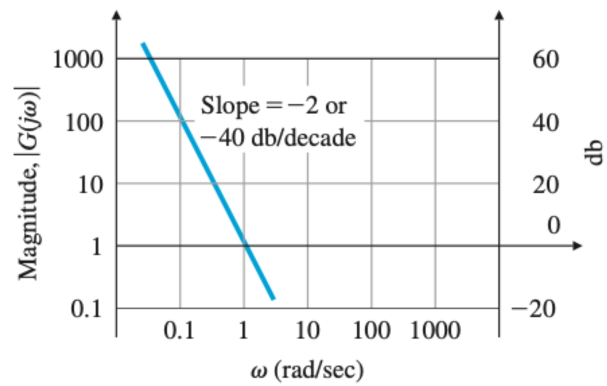


Figure 6.46
Magnitude of the spacecraft's frequency response

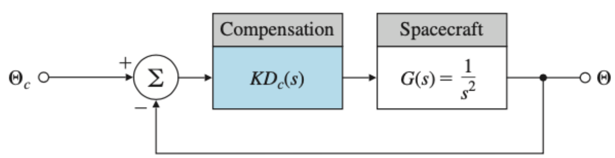


Figure 6.45
Spacecraft attitude-control system

$$KD_c(s)G(s) = \frac{K(T_D s + 1)}{s^2}$$

$$|KD_c(j\omega)G(j\omega)| = \frac{K\sqrt{T_D^2\omega^2 + 1}}{\omega^2}$$

$$\angle KD_c(j\omega)G(j\omega) = \tan^{-1} T_D\omega - 180°$$

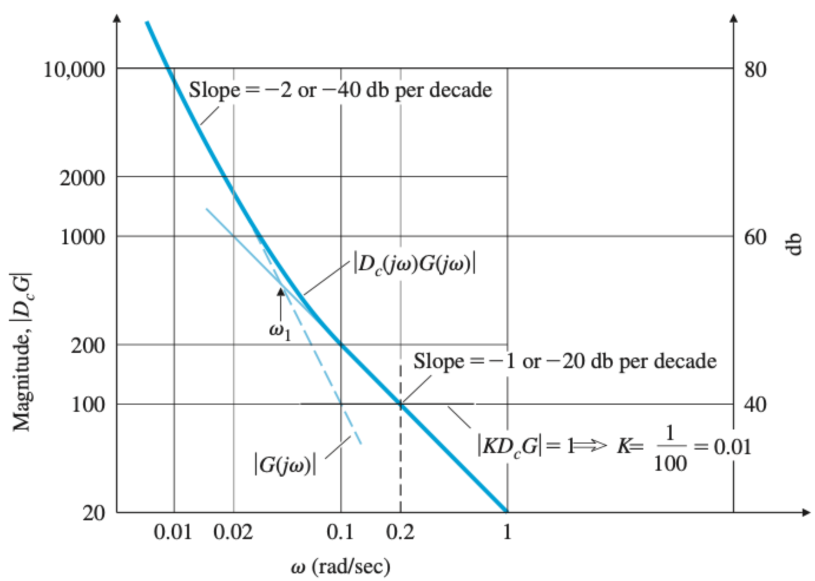
1. Phase margin at the gain crossover frequency

$$PM = 180^\circ + \tan^{-1} T_D\omega_g - 180^\circ = \tan^{-1} T_D\omega_g = 75^\circ \quad \rightarrow \quad T_D = \frac{1}{\omega_g}\tan 75^\circ = 18.66$$

2. Gain crossover frequency ω_g

$$\frac{K\sqrt{T_D^2\omega_g^2+1}}{\omega_g^2} = 1 \quad \rightarrow \quad K = \frac{\omega_g^2}{\sqrt{T_D^2\omega_g^2+1}} = 0.01$$

Figure 6.47
Compensated open-loop transfer function



6 Closed-Loop Frequency Response

- Consider a system in which $|KG(j\omega)|$ shows the typical behavior

$$|KG(j\omega)| \gg 1 \qquad \text{for} \quad \omega \ll \omega_c$$
$$|KG(j\omega)| \ll 1 \qquad \text{for} \quad \omega \gg \omega_c$$

where ω_c is the crossover frequency. The magnitude of closed-loop frequency response is approximated by

$$|T(j\omega)| = \left|\frac{KG(j\omega)}{1+KG(j\omega)}\right| \approx \begin{cases} 1 & \text{for} \quad \omega \ll \omega_c \\ |KG(j\omega)| & \text{for} \quad \omega \gg \omega_c \end{cases}$$

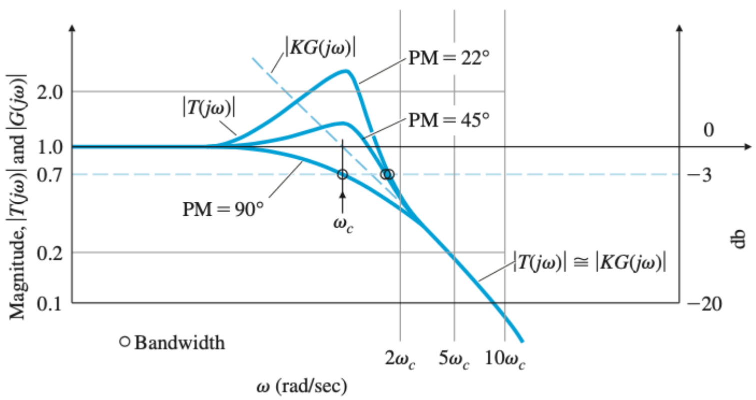


Figure 6.50
Closed-loop bandwidth with respect to PM

- The graph shows that the bandwidth for smaller values of PM is typically somewhat greater than ω_c, though usually it is less than $2\omega_c$; thus

$$\omega_c \le \omega_{BW} \le 2\omega_c$$