5 Effects of Zeros and Additional Poles

1. When the target performance is satisfied with the pole locations change
   - If a system response is too slow, we must raise the natural frequency
   - If the transient has too much overshoot, then the damping needs to be increased
   - If the transient persists too long, the poles need to be moved to the left in the s-plane.
2. When the target performance is not satisfied with the pole locations change, we can add new zero and additional pole.
   - To illustrate the effect of zero addition, consider the following two examples:

$$
\begin{aligned}
H_1(s) &= \frac{2}{(s+1)(s+2)} & H_2(s) &= \frac{2(s+1.1)}{1.1(s+1)(s+2)} \\
&= \frac{2}{s+1} - \frac{2}{s+2} & &= \frac{0.18}{s+1} + \frac{1.64}{s+2} \\
h_1(t) &= 2e^{-t} - 2e^{-2t} & h_2(t) &= 0.18e^{-t} + 1.64e^{-2t} \quad \text{for} \quad t \geq 0
\end{aligned}
$$

where addition of zero $(s+1.1)$ brought the dramatic reduction of effect of pole $(s+1)$.

- Consider a typical 2nd-order system and its zero addition:

$$H_1(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad\qquad H_2(s) = \frac{\omega_n^2\left(\frac{s}{\alpha\zeta\omega_n} + 1\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

  The zero is located at $s = -\alpha\zeta\omega_n = -\alpha\sigma$.

- If α is large, the zero will be far removed from the poles and the zero will have little effect on the response.
- If $\alpha \approx 1$, the value of the zero will be close to that of the real part of the poles and can be expected to have a substantial influence on the response.
- When $\zeta = 0.5$ and $\zeta = 0.707$, the step response curves with several values of α are plotted. The major effect of the zero is
  - to increase the overshoot M_p
  - to reduce the rise time t_r
  - whereas it has very little influence on the settling time t_s

Figure 3.27 Plots of the step response of a second-order system with a zero (ζ = 0.5)

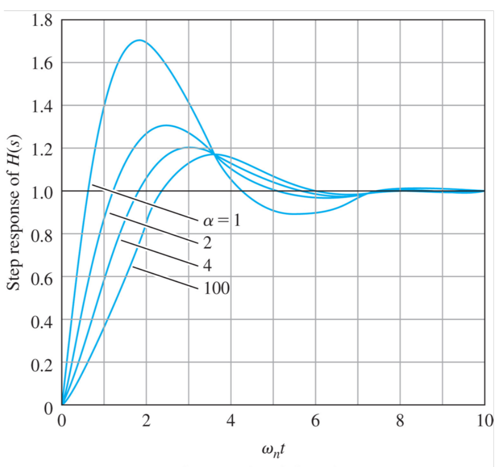


Figure 3.28 Plots of the step response of a second-order system with a zero (ζ = 0.707)

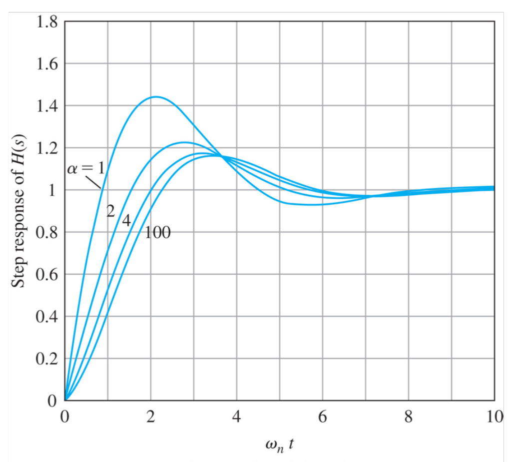
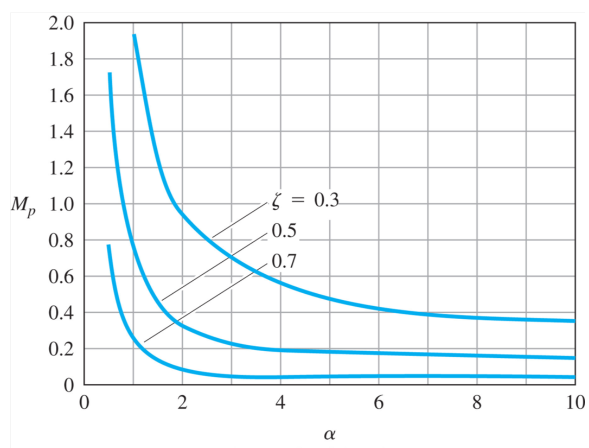


Figure 3.29 Plot of overshoot M_p as a function of normalized zero location α. At α = 1, the real part of the zero equals the real part of the poles



3. Take simple example ($\omega_n = 1$)

$$
\begin{aligned}
H(s) &= \frac{\frac{s}{\alpha\zeta} + 1}{s^2 + 2\zeta s + 1} \\
&= \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1} \\
&= H_0(s) + H_d(s)
\end{aligned}
$$

Now if we apply the unit-step input to above system, then we have

$$
\begin{aligned}
y(t) &= y_0(t) + y_d(t) \\
&= y_0(t) + \frac{1}{\alpha\zeta}\dot{y}_0(t)
\end{aligned}
$$

where $y_0(t) = \mathcal{L}^{-1}[H_0(s)\frac{1}{s}]$. It is noted that the zero increases the overshoot.

4. When $\alpha < 0$, the zero is in the RHP where $s > 0$. This is typically called as RHP zero and is sometimes referred to as a non-minimum phase zero. Also it is noted that "initial undershoot" can be occurred.

Figure 3.30 Second-order step responses $y(t)$ of the transfer functions $H(s)$, $H_0(s)$, and $H_d(s)$

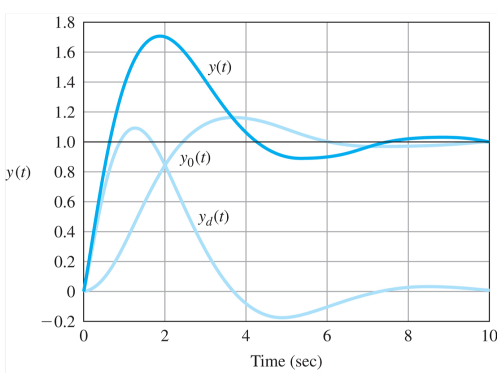
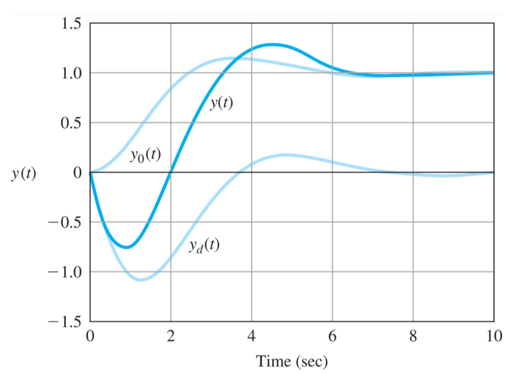


Figure 3.31 Step responses $y(t)$ of a second-order system with a zero in the RHP: a nonminimum-phase system



5. (Example 3.28, Effect of the proximity of the Zero to the Pole locations on the TF) Consider the 2nd-order system with a finite zero and unity DC gain and determine the effect of the zero location on the unit-step response when $z = \{1, 2, 3, 4, 5, 6\}$

$$H(s) = \frac{24}{z}\frac{(s+z)}{(s+4)(s+6)}$$

The unit-step response is obtained as

$$\begin{aligned}
Y(s) &= H(s)\frac{1}{s} \\
&= \frac{24}{z}\frac{(s+z)}{s(s+4)(s+6)} \\
&= \frac{24}{s(s+4)(s+6)} + \frac{1}{z}\frac{24s}{s(s+4)(s+6)} \\
y(t) &= y_0(t) + \frac{1}{z}\dot{y}_0(t)
\end{aligned}$$

For $y_0(t)$

$$\begin{aligned}
Y_0(s) &= \frac{24}{s(s+4)(s+6)} \\
&= \frac{1}{s} - \frac{3}{s+4} + \frac{2}{s+6} \\
y_0(t) &= 1 - 3e^{-4t} + 2e^{-6t} \quad \textbf{for} \quad t \geq 0
\end{aligned}$$

For $\dot{y}_0(t)$

$$\dot{y}_0(t) = 12e^{-4t} - 12e^{-6t} \quad \textbf{for} \quad t \geq 0$$

Thus the complete solution is

$$y(t) = 1 + \left(\frac{12}{z} - 3\right) e^{-4t} + \left(2 - \frac{12}{z}\right) e^{-6t} \qquad \textbf{for} \quad t \geq 0$$

For $z = 5$, where the zero is located between two poles, there is no overshoot. This is generally the case because the zero effectively compensates for the effect of the second pole, rendering the system as first-order at any given frequency.

Figure 3.32 Effect of zero on transient response

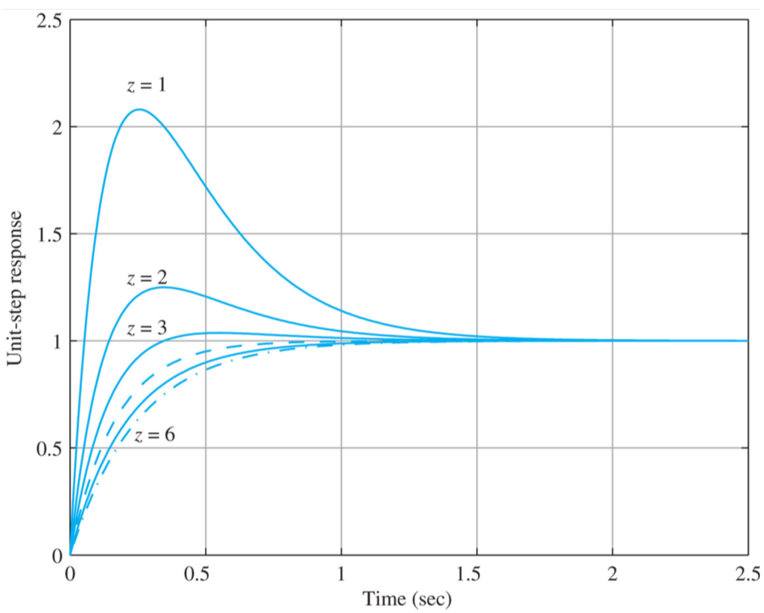


Figure 3.33 Locations of complex zeros

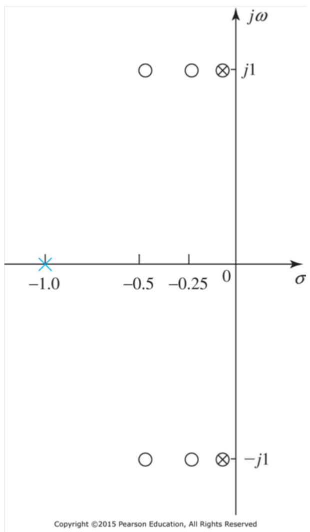
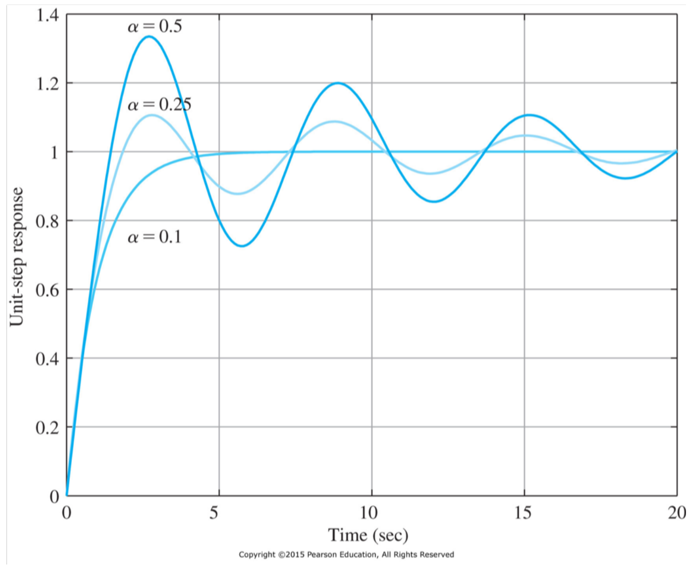


Figure 3.34 Effect of complex zeros on transient response



6. (Example 3.29, Effect of the proximity of the Complex Zeros to the Lightly Damped Poles using MATLAB) Consider the third-order feedback system with a pair of lightly damped poles and a pair of complex zeros with the TF. Determine the effect of the complex zero locations ($s = -\alpha \pm j\beta$) on the unit-step response of the system for three different zero locations $(\alpha, \beta) = (0.1, 1.0), (0.25, 1.0), (0.5, 1.0)$

$$H(s) = \frac{(s+\alpha)^2 + \beta^2}{(s+1)[(s+0.1)^2+1]}$$

We plot the three unit-step responses using MATLAB. Placing the complex zeros near the locations of the lightly damped poles may provide sufficient improvement in step response performance.

7. (Example 3.30, Aircraft Response using MATLAB) The TF b/w elevator and altitude of the Boeing 747 aircraft can be approximated as

$$\frac{h(s)}{\delta_e(s)} = \frac{30(s-6)}{s(s^2+4s+13)}$$

- Unit-step response is shown in Fig. 3.35 (initial undershoot due to non-minimum phase zero)
- Several specifications

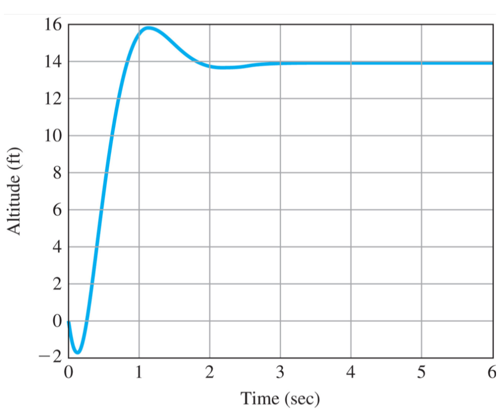
$$h(\infty) = \lim_{s\to 0} s\frac{30(s-6)}{s(s^2+4s+13)}\frac{-1}{s} = 13.8$$

$$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{13}} = 0.5[s] \qquad \zeta = 0.55 \quad \text{from} \quad 2\zeta\omega_n = 4$$

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.14$$

$$t_s = \frac{4.6}{\sigma} = 2.3 \qquad \text{1\% criterion}$$

Figure 3.35 Response of an airplane's altitude to an impulsive elevator input

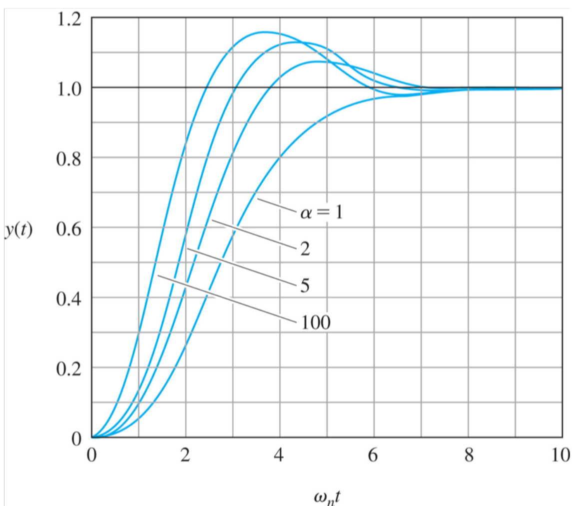


8. Effect of an extra pole: consider the TF as follow:

$$H(s) = \frac{\omega_n^2}{\left(\frac{s}{\alpha\zeta\omega_n} + 1\right)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

where $s = -\alpha\zeta\omega_n$ is additional pole location. The plot of the step responses with $\alpha = 1, 2, 5, 100$ is given for $\zeta = 0.5$. Major effect is to increase the rise time.

Figure 3.36 Step responses for several third-order systems with $\zeta = 0.5$



9. Effect of Pole-Zero Patterns on Dynamic Response

a) For 2nd-order system with no finite zeros, transient response parameters are approximated:

$$\text{rise time} \qquad t_r \approx \frac{1.8}{\omega_n}$$

$$\text{overshoot} \qquad M_p \approx \begin{cases} 5\% & \zeta = 0.7 \\ 16\% & \zeta = 0.5 \\ 35\% & \zeta = 0.3 \end{cases}$$

$$\text{settling time} \qquad t_s \approx \frac{4.6}{\sigma} \qquad \text{1\% criterion}$$

b) A zero in the left half plane (LHP) will increase the overshoot if the zero is within a factor of 4 of the real part of the complex poles

c) A zero in the RHP will depress the overshoot (but it may have undershoot)

d) An additional pole in the LHP will increase the rise time significantly if the extra pole is within a factor of 4 of the real part of the complex pole.

Figure 3.29 Plot of overshoot M_p as a function of normalized zero location α. At $\alpha = 1$, the real part of the zero equals the real part of the poles

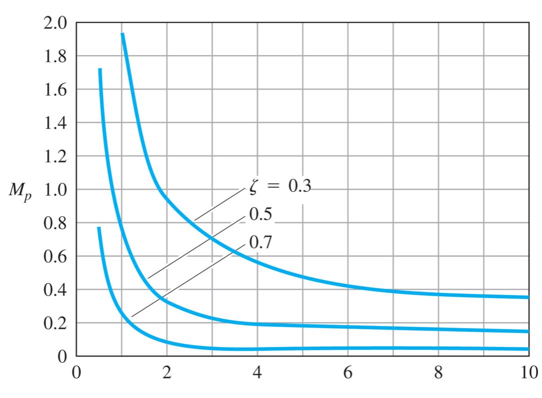


Figure 3.37 Normalized rise time for several locations of an additional pole