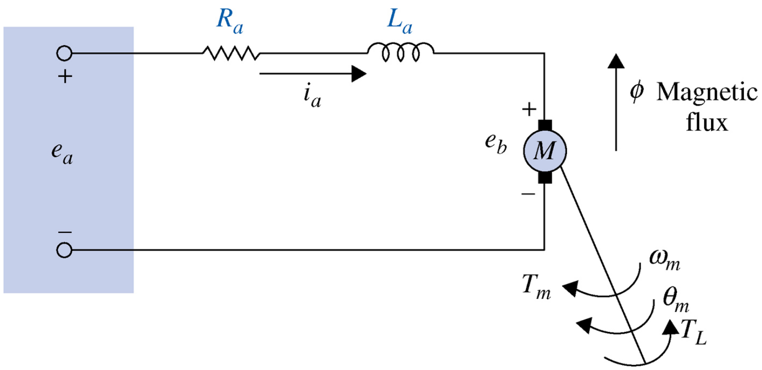
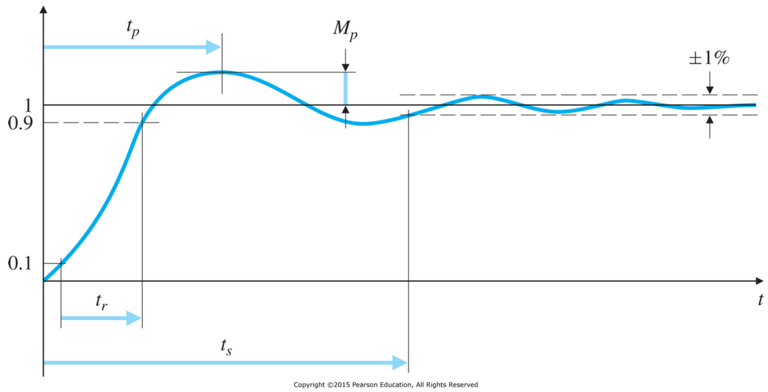
(Question: 영구자석형 dc 모터의 모델링 : Mathematical Modeling of PM DC Motors)



4 Time-Domain Specifications



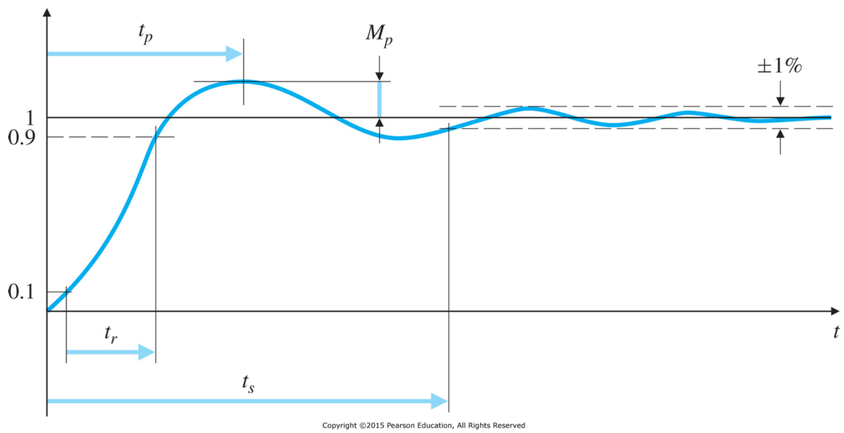
Consider a typical unit step response

- The rise time t_r is the time it takes the system to reach the vicinity of its new set point
- The settling time t_s is the time it takes the system transients to decay
- The overshoot M_p is the maximum amount that system overshoots its final value divided by its final value (and is often expressed as a percentage)
- The peak time t_p is the time it takes the system to reach the maximum overshoot point.

1. Rise Time

   - For second-order system, a rise time can be found by approximating the curve at $\zeta = 0.5$. The rise time from $y = 0.1$ to $y = 0.9$ is

$$\therefore \quad t_r \approx \frac{1.8}{\omega_n}$$



2. Overshoot and Peak Time

- From the step response for a typical second-order system, we have

$$\begin{aligned} Y(s) &= H(s)\frac{1}{s} \\ &= \frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}\frac{1}{s} \\ &= \frac{\omega_n^2}{(s+\sigma)^2+\omega_d^2}\frac{1}{s} \\ &= \frac{C_1}{s}+\frac{C_2(s+\sigma)+C_3\omega_d}{(s+\sigma)^2+\omega_d^2} \end{aligned}$$

where $C_1 = 1$, $C_2 = -1$, and $C_3 = -\frac{\zeta}{\sqrt{1-\zeta^2}}$. Thus we have

$$
\begin{aligned}
Y(s) &= \frac{1}{s} - \frac{(s+\sigma)}{(s+\sigma)^2+\omega_d^2} - \frac{\zeta}{\sqrt{1-\zeta^2}}\frac{\omega_d}{(s+\sigma)^2+\omega_d^2} \\
y(t) &= 1 - e^{-\sigma t}\cos\omega_d t - \frac{\zeta}{\sqrt{1-\zeta^2}} e^{-\sigma t}\sin\omega_d t \qquad \textbf{for } t \geq 0 \\
&= 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)
\end{aligned}
$$

- When $y(t)$ reaches its maximum value, its derivative will be zero:

$$
\begin{aligned}
\dot{y}(t) &= \sigma e^{-\sigma t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) - e^{-\sigma t}\left(-\omega_d\sin\omega_d t + \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t\right) = 0 \\
&= e^{-\sigma t}\left(\frac{\zeta\sigma}{\sqrt{1-\zeta^2}} + \omega_d\right)\sin\omega_d t = 0
\end{aligned}
$$

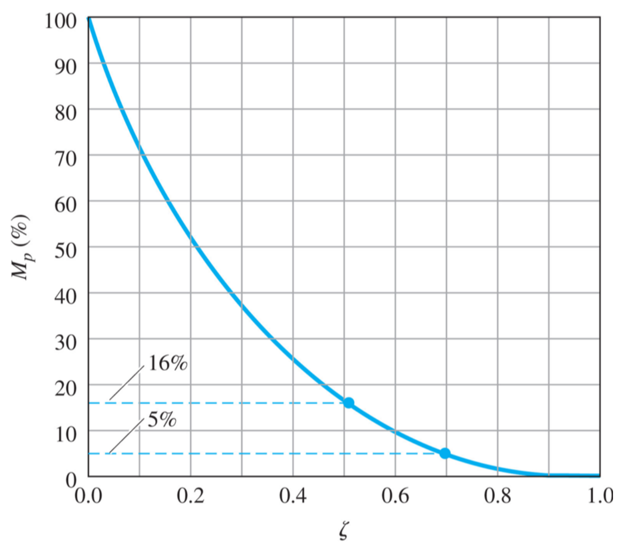
This occurs when $\sin\omega_d t = 0$, so the peak time is obtained as

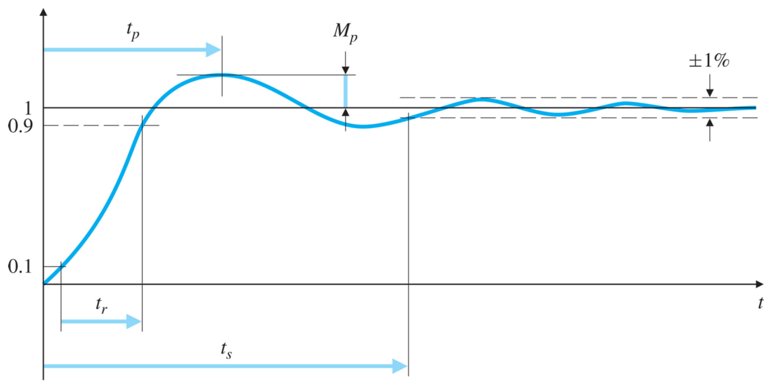
$$
\omega_d t_p = \pi \qquad \rightarrow \qquad \therefore \quad t_p = \frac{\pi}{\omega_d}
$$

- The maximum overshoot at $t = t_p$ is

$$\begin{aligned} y(t_p) &= 1 - e^{-\sigma t_p}\left(\cos\omega_d t_p + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_p\right) \\ &= 1 - e^{-\sigma t_p}\left(\cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi\right) \\ &= 1 + e^{-\sigma t_p} \\ &= 1 + e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \quad \rightarrow \quad \therefore \quad M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \end{aligned}$$

where it is noted that the maximum overshoot is only function of damping ratio, irrespective of the natural frequency. For example, $M_p = 0.16$ (16% overshoot) for $\zeta = 0.5$, $M_p = 0.1$ (10% overshoot) for $\zeta = 0.6$, and $M_p = 0.05$ (5% overshoot) for $\zeta = 0.7$





3. Settling Time

- Typical step response

$$
\begin{aligned}
Y(s) &= 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) \qquad \text{for } t \geq 0 \\
&= 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right) \\
&= 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos\left(\omega_d t - \tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}}\right)
\end{aligned}
$$

where it is noted that

$$
\begin{aligned}
A\sin\alpha + B\cos\alpha &= \sqrt{A^2+B^2}\,\sin(\alpha+\beta) \qquad \beta = \tan^{-1}\frac{B}{A} \\
&= \sqrt{A^2+B^2}\,\cos(\alpha-\gamma) \qquad \gamma = \tan^{-1}\frac{A}{B}
\end{aligned}
$$

- Envelope exponentials are

$$1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}} \quad \text{and} \quad 1 + \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}$$

Indeed, if the settling time is specified as c_{ts} ($c_{ts} = 0.05$ for 5%, $c_{ts} = 0.03$ for 3%, and $c_{ts} = 0.01$ for 1%), then we have

$$1 - \frac{e^{-\sigma t_s}}{\sqrt{1-\zeta^2}} = 1 - c_{ts}$$
$$1 + \frac{e^{-\sigma t_s}}{\sqrt{1-\zeta^2}} = 1 + c_{ts}$$

Thus we have an exact settling time as follow:

$$\frac{e^{-\sigma t_s}}{\sqrt{1-\zeta^2}} = c_{ts} \qquad \rightarrow \qquad \therefore \qquad t_s = -\frac{1}{\zeta\omega_n} \ln c_{ts}\sqrt{1-\zeta}$$

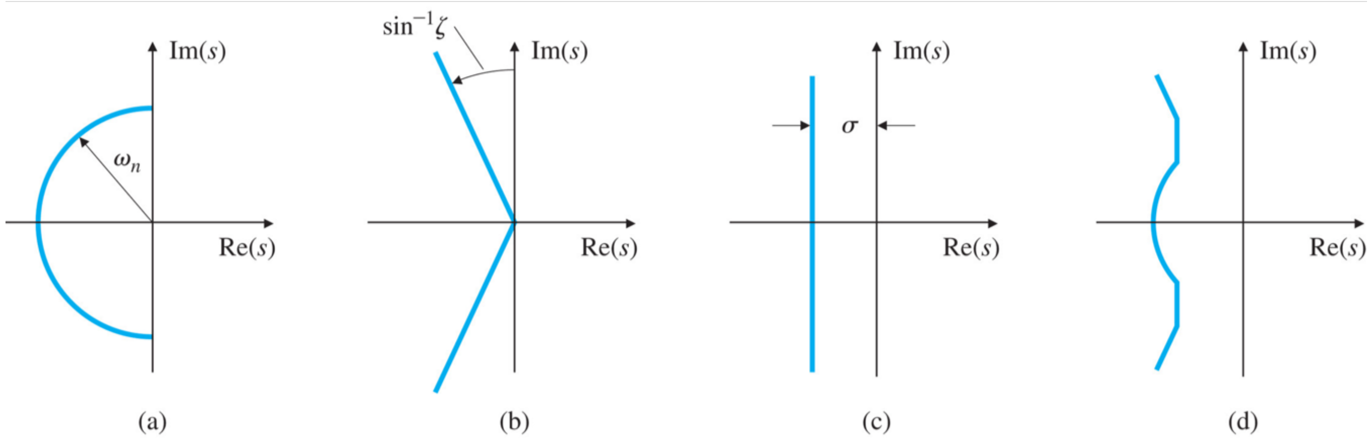
- On the other hand, when the decaying exponential reaches 1%, we can get the "approximate settling time (1% criterion)" as follow:

$$e^{-\sigma t_s} = e^{-\zeta\omega_n t_s} = 0.01 \qquad \rightarrow \qquad \therefore \qquad t_s \approx \frac{-\ln 0.01}{\zeta\omega_n} = \frac{4.6}{\sigma} = \frac{4.6}{\zeta\omega_n}$$

4. Design Synthesis: for specified values of t_r, M_p and t_s,

$$\omega_n \geq \frac{1.8}{t_r} \quad \text{and} \quad \zeta \geq \zeta(M_p) \;\; \text{using } M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \quad \text{and} \quad \sigma \geq \frac{4.6}{t_s} \;\; \text{or} \;\; \sigma \geq -\frac{1}{t_s}\ln c_{ts}\sqrt{1-\zeta}$$

These inequalities can be graphed in the s-plane and they will be used in later chapters to guide the selections of pole and zero locations to meet control system specifications for dynamic response.



5. For a first-order system and its step response,

$$H(s) = \frac{\sigma}{s+\sigma}$$

$$Y(s) = H(s)\frac{1}{s} = \frac{\sigma}{s(s+\sigma)} = \frac{1}{s} - \frac{1}{s+\sigma}$$

$$y(t) = 1 - e^{-\sigma t} \qquad \textbf{for } t \geq 0$$

Specifications for 1st-order system with time constant $\tau = \frac{1}{\sigma}$

$$t_s = \frac{4.6}{\sigma} = 4.6\tau \qquad \textbf{and} \qquad t_r = \frac{\ln 0.9 - \ln 0.1}{\sigma} = \frac{2.2}{\sigma} = 2.2\tau \qquad \textbf{and} \qquad M_p = 0$$

6. (Example 3.27, Transformation of the Specifications to the s-Plane) Find the allowable region in the s-plane when $t_r \leq 0.6[s]$, $M_p \leq 0.1$ and $t_s \leq 3[s]$

$$\omega_n \geq \frac{1.8}{t_r} = 3.0 \quad \text{and} \quad \zeta \geq 0.6 \quad \text{and} \quad \sigma \geq \frac{4.6}{3} = 1.5$$

For the allowable region, see the Fig. 3.26