(DOB) 1. Overview of Disturbance Observer

1. Disturbances widely exist in modern industrial control systems and bring *adverse effects* to the performance of control systems.

2. Therefore, *disturbance rejection* is one of the key objectives in controller design.

3. The disturbances refer to not only the disturbances from the *external* environment of a control system but also *uncertainties* from the controlled plant including unmodeled dynamics, parameter perturbations, and nonlinear couplings of multivariable systems.

4. DOB is utilized for
    - the process control community,
    - the electrical control community,
    - the mechanical control community,
    - the aeronautic and astronautic engineering community

5. Several temninologies
    - passive antidisturbance control (PADC)
    - active antidisturbance control (AADC)
    - feedforward control (FC)
    - *disturbance observer-based control* (DOBC)

(DOB) 1.2 Motivations

1. We start with a *simple example* to illustrate the major motivation as the following system:

$$\dot{x} = -ax + u + d$$
$$y = x$$

   where u the control input, x the state, y the interested controlled output, a the system parameter, and d the disturbance.

2. Let y_r the *desired* value that the output is expected to achieve, which is usually called *setpoint* or object value, that is, $\dot{y}_r = 0$.

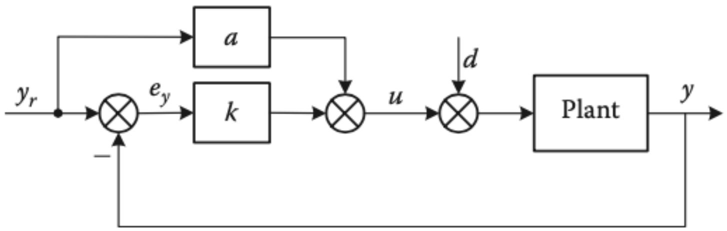
3. Defining the *tracking error* variable of the system as $e_y = y_r - y$, the error is given by

$$\begin{aligned} e_y &= y_r - y \\ \dot{e}_y &= \dot{y}_r - \dot{y} = 0 - [-ay + u + d] = -a(y_r - y) - u - d + ay_r \\ &= -ae_y - u - d + ay_r \end{aligned}$$

4. The control object here is to design a control law u in terms of the tracking error e_y and the setpoint y_r, that is, $u = u(e_y, y_r)$, such that the actual output y achieves its desired setpoint y_r, that is, the tracking error $e_y \to 0$ as $t \to \infty$

   - High-Gain Control
   - Integral Control
   - DOBC

(DOB) 1.2.1 High-Gain Control



1. Proportional control is usually utilized to realize the control object,

$$u = ke_y + ay_r$$

2. The closed-loop system under the *proportional control* is described by

$$\dot{e}_y = -ae_y - u - d + ay_r = -ae_y - (ke_y + ay_r) - d + ay_r = -(a+k)e_y - d$$

where k is the *proportional gain* to be designed.

3. The *solution* is obtained by multiplying $e^{(a+k)t} > 0$ and then by integrating from 0 to t:

$$\begin{aligned}
\dot{e}_y + (a+k)e_y = -d \quad &\rightarrow \quad \dot{e}_y e^{(a+k)t} + (a+k)e_y e^{(a+k)t} = -de^{(a+k)t} \\
\frac{d}{dt}[e_y(t)e^{(a+k)t}] = -d(t)e^{(a+k)t} \quad &\rightarrow \quad \int_0^t \frac{d}{d\tau} e_y(\tau)e^{(a+k)\tau} d\tau = \int_0^t -d(\tau)e^{(a+k)\tau} d\tau \\
e_y(t)e^{(a+k)t} - e_y(0) = -\int_0^t d(\tau)e^{(a+k)\tau} d\tau \quad &\rightarrow \quad \therefore \quad e_y(t) = e^{-(a+k)t}e_y(0) - \int_0^t e^{-(a+k)(t-\tau)} d(\tau) d\tau
\end{aligned}$$

4. Suppose that the disturbance in the system is *bounded* and satisfies $|d(t)| < d^*$

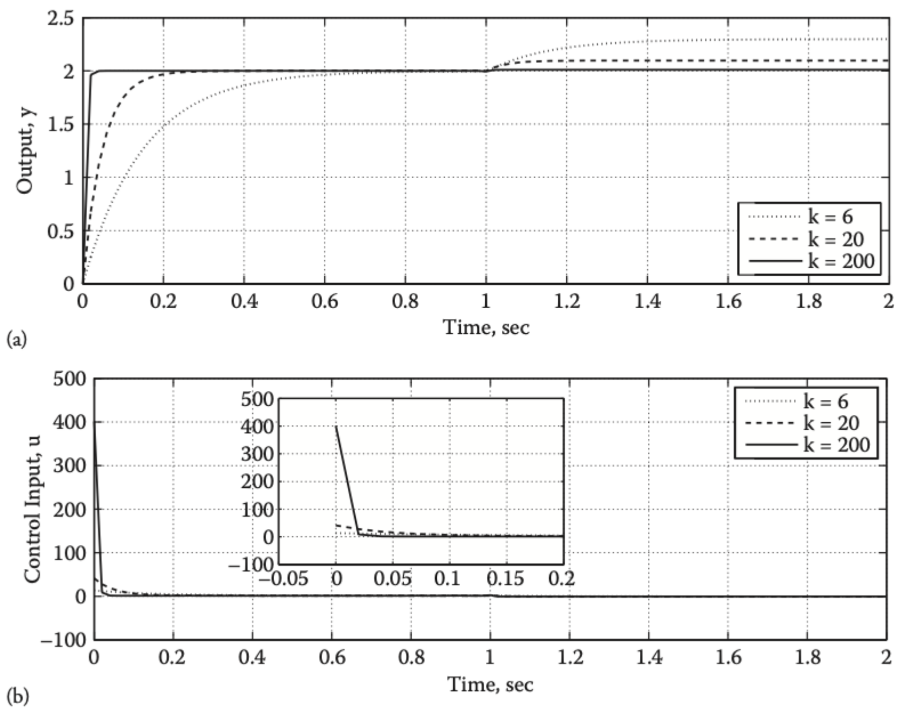
$$
\begin{aligned}
|e_y(t)| &\le e^{-(a+k)t}|e_y(0)| + \int_0^t e^{-(a+k)(t-\tau)}|d(\tau)|d\tau \\
&\le e^{-(a+k)t}|e_y(0)| + d^* \int_0^t e^{-(a+k)(t-\tau)}d\tau \\
&= e^{-(a+k)t}|e_y(0)| + d^* \left[\frac{e^{-(a+k)(t-\tau)}}{a+k}\right]_0^t \\
&= e^{-(a+k)t}|e_y(0)| + \frac{d^*}{a+k}[1 - e^{-(a+k)t}]
\end{aligned}
$$

5. Taking the *final value* of the solution, we have

$$
\therefore \quad |e_y(\infty)| \le \frac{d^*}{a+k}
$$

6. It is concluded that proportional control cannot completely remove the effects caused by disturbance (even a constant one) from the systems.

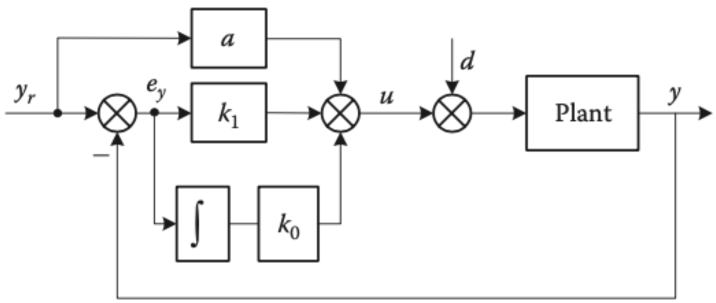
7. In the presence of disturbance, to maintain a smaller tracking offset, a higher control gain k has to be designed to suppress the disturbance.



8. Simulation is conducted with a step disturbance $d = 2$ applied at $t = 1[s]$.

   - As shown in the figure, a higher control gain usually generates a faster tracking response and a smaller offset in the presence of disturbances.
   - The results of this simulation scenario demonstrate that there exists a *trade-off* between disturbance rejection and reasonable control energy for the high gain control method.

(DOB) 1.2.2 Integral Control (+ Proportional Control)



1. In practical engineering systems, the integral control action is always employed *to remove the offset* in the presence of disturbances and uncertainties.

2. The integral control law is usually designed as

$$u = k_1 e_y + k_0 \int_0^t e_y(\tau) d\tau + a y_r$$

   where k_1 and k_0 are the *proportional and integral gains* to be designed, respectively.

3. The closed-loop system is expressed by

$$\dot{e}_y = -a e_y - u - d + a y_r = -a e_y - \left[k_1 e_y + k_0 \int_0^t e_y(\tau) d\tau + a y_r \right] - d + a y_r = -(a + k_1) e_y - k_0 \int_0^t e_y(\tau) d\tau - d$$

4. Taking time derivatives of both sides gives

$$\therefore \quad \ddot{e}_y + (a + k_1) \dot{e}_y + k_0 e_y + \dot{d} = 0$$

5. Let $x = [e_y, \dot{e}_y]^T$, we can get *error state-space* representation as follow:

$$\frac{d}{dt}\begin{bmatrix} e_y \\ \dot{e}_y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_0 & -(a+k_1) \end{bmatrix}\begin{bmatrix} e_y \\ \dot{e}_y \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}\dot{d} \qquad \rightarrow \qquad \dot{x} = Ax + B\dot{d}$$

6. In the presence of constant disturbance, i.e., $\dot{d}(t) = 0$, if A is a Hurwitz matrix, it is derived from above equation that $x(\infty) = [0, 0]^T$. This means that the integral control can finally remove the effects caused by constant disturbance from the system.

7. However, the integral control *cannot remove the effects caused by non-constant* disturbances, such as harmonic ones.

8. The solution vector is obtained by multiplying the matrix exponential $e^{-At} > 0$ and then integrating from 0 to t

$$\begin{aligned}
\dot{x} - Ax = B\dot{d} \qquad &\rightarrow \qquad \dot{x}e^{-At} - Axe^{-At} = B\dot{d}e^{-At} \\
\frac{d}{dt}[xe^{-At}] = B\dot{d}e^{-At} \qquad &\rightarrow \qquad \int_0^t \frac{d}{d\tau}x(\tau)e^{-A\tau}d\tau = B\int_0^t \dot{d}(\tau)e^{-A\tau}d\tau \\
x(t)e^{-At} - x(0) = B\int_0^t \dot{d}(\tau)e^{-A\tau}d\tau \qquad &\rightarrow \qquad \therefore\ x(t) = e^{At}x(0) + B\int_0^t e^{A(t-\tau)}\dot{d}(\tau)d\tau
\end{aligned}$$

9. Suppose that the disturbance is *non-constant, but bounded and with a bounded derivative*, i.e., $|\dot{d}(t)| < \dot{d}^*$.

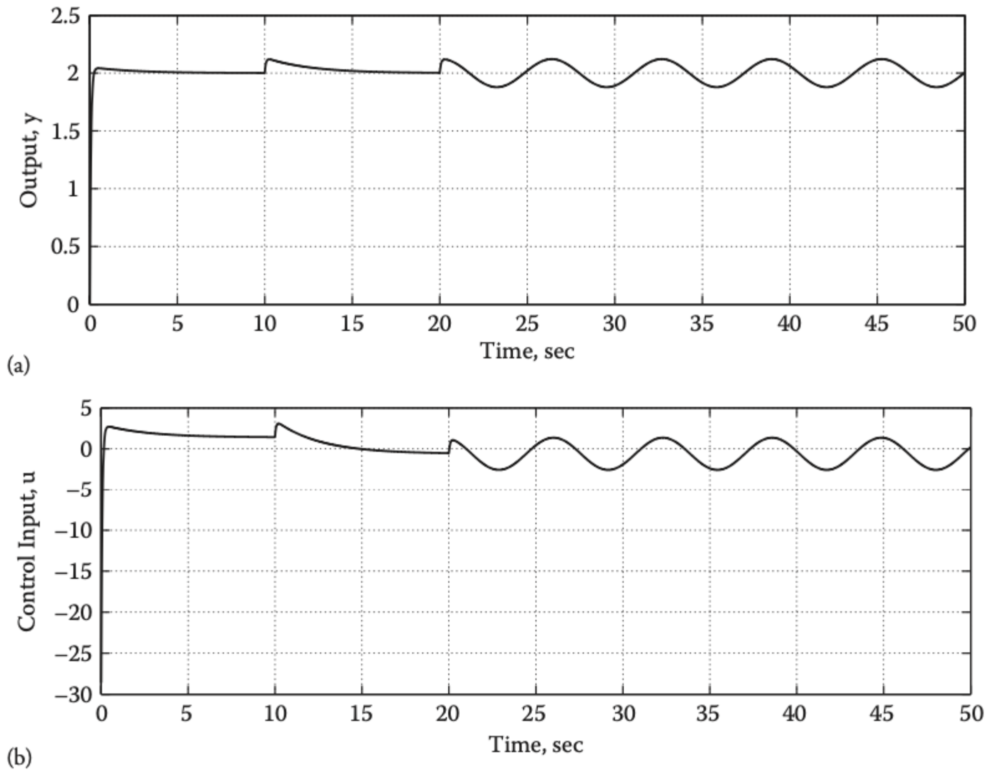
$$
\begin{aligned}
|x(t)| &\leq e^{At}|x(0)| + |B| \int_0^t e^{A(t-\tau)} |\dot{d}(\tau)| d\tau \\
&\leq e^{At}|x(0)| + |B|\dot{d}^* \int_0^t e^{A(t-\tau)} d\tau \\
&= e^{At}|x(0)| + |B|\dot{d}^* \left[-A^{-1} e^{A(t-\tau)} \right]_0^t \\
&\leq e^{At}|x(0)| + |A^{-1}B|\dot{d}^* |e^{At} - I|
\end{aligned}
$$

10. Taking limits of both sides yields, since $\lim_{t\to\infty} e^{At} = 0_{2\times 2}$ in the case of a stable system,

$$
\therefore \quad |x(\infty)| \leq |A^{-1}B|\dot{d}^*
$$

11. It is concluded from above relations that integral control can only *remove the offset caused by constant* disturbance. However, in the presence of non-constant disturbances, the integral control may result in a steady-state tracking error.

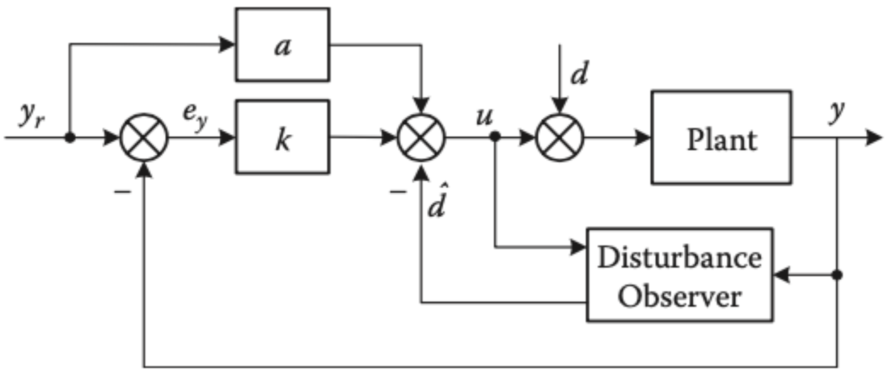
12. The integral action always causes undesirable transient control performances, such as *larger overshoot*.



13. Step disturbance $d(t) = 2$ is applied at $t = 10[s]$ and the harmonic disturbance $d(t) = 2 + 2\sin(t)$ is added at $t = 20[s]$.

- In the presence of constant disturbance, the integral control can remove the offset of the tracking error. However, the integral control is *unavailable to reject the harmonic disturbance*.
- The results of this simulation scenario demonstrate that there is a *trade-off* between disturbance rejection and tracking performance for the integral control method.

(DOB) 1.2.3 Disturbance Observer-based Control (DOBC)



1. The high gain control and integral control can be recognized as a PADC method.

2. Different from the high gain control and integral control approaches, DOBC provides an *active and effective way to handle disturbances and improve robustness* of the closed-loop system, which is always considered as an AADC method.

3. The DOBC law is usually designed as

$$u = ke_y - \hat{d} + ay_r$$

   where k is the control gain, and $\hat{d}$ is the *disturbance estimation* by a disturbance observer.

4. The closed-loop system is described by

$$\dot{e}_y = -ae_y - u - d + ay_r = -ae_y - [ke_y - \hat{d} + ay_r] - d + ay_r = -(a + k)e_y + e_d$$

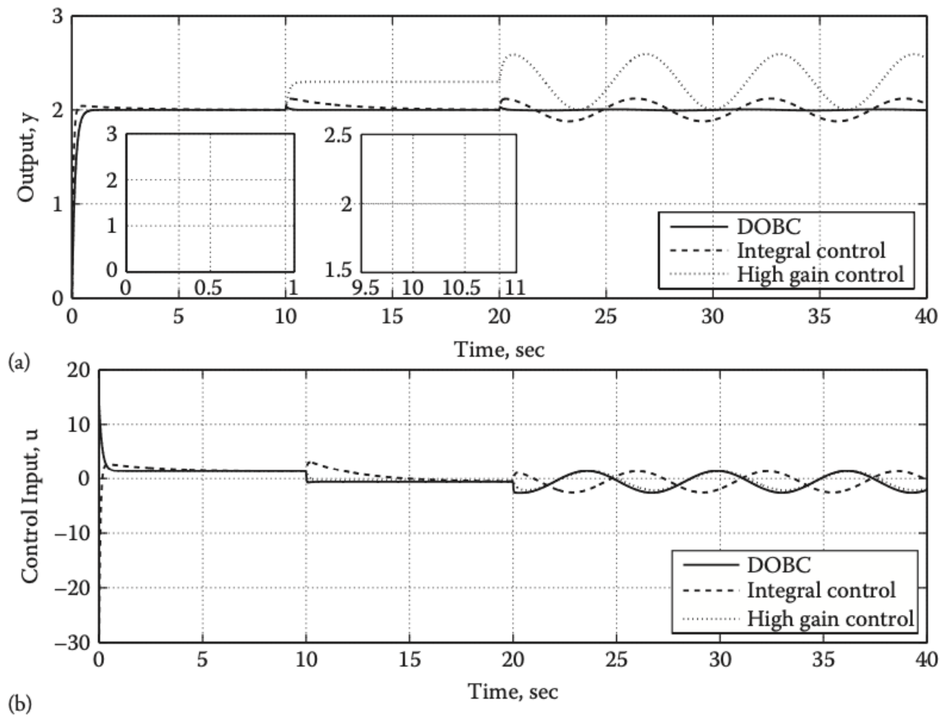
   where $e_d = \hat{d} - d$ is the *disturbance estimation error*.

5. By designing an appropriate disturbance observer, the disturbance estimation error e_d can be usually governed by

$$\dot{e}_d = f(e_d)$$

which is globally asymptotically stable.
The DO design would be one of the main targets for the DOBC.

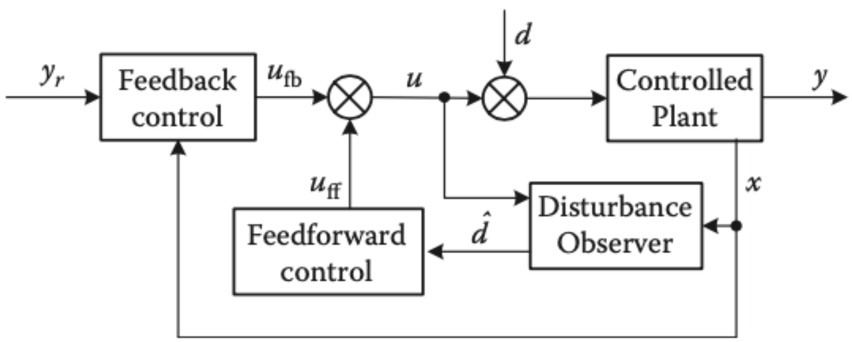
6. Compared with the integral control method, the DOBC can compensate not only *constant* disturbances but also *many other types of disturbances*, such as harmonic ones, as long as they can be accurately estimated by a disturbance observer.



7. Step disturbance $d(t) = 2$ at $t = 10[s]$, and Harmonic disturbance $d(t) = 2 + 2\sin(t)$ at $t = 20[s]$.

- As shown in the figure, the DOBC method could reject both constant and harmonic disturbances more promptly as compared with both the proportional control and the integral control methods, and no excessive control energy is required for disturbance rejection.
- The response curves under the DOBC method are overlapped with those of the baseline proportional control method, which is called *nominal performance recovery*.
- The reason for this lies in that the disturbance observer-based compensation serves like a *patch to the baseline proportional control*.
- Generally speaking, DOBC could achieve a *good disturbance rejection* without scarifying the nominal performance.

(DOB) 1.3 Basic DOB Framework

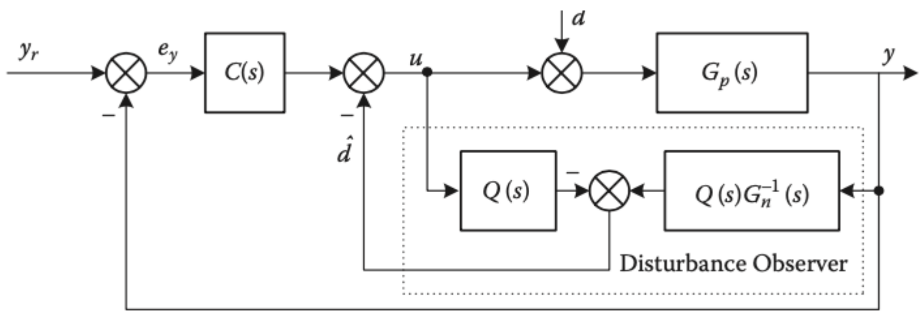


1. A basic framework of disturbance observer based control method is shown in the figure.
2. The composite controller consists of two parts: a *feedback* control part and a *feedforward* control part based on a disturbance observer.
    - The feedback control is generally employed *for tracking and stabilization of the nominal dynamics* of the controlled plant. In this stage, the disturbances and uncertainties are not necessarily required to be considered.
    - The disturbances and uncertainties on controlled plant are estimated by a *disturbance observer* and then compensated by a feedforward control.
    - The major merit of such design lies in that the feedback control and feedforward designs satisfy the so-called *separation principle*, that is, the *tracking control performance and the disturbance rejection performance* can be achieved by adjusting the feedback and feedforward controllers, respectively.

3. Promising features of the DOB:

   - Faster responses in handling the disturbances:
     - As compared with the PADC method, DOBC always achieves a *faster dynamic response* in handling disturbances.
     - The reason is that DOBC provides a *feedforward compensation* term to directly counteract the disturbances in the control systems, while the PADC only rejects the disturbances by *passive feedback* regulation.
   - Patch features:
     - The disturbance feedforward compensation term can be considered as a *patch* to the existing feedback control.
     - The benefit of this is that there is *no change to the baseline control*, which may have been widely used and developed for many years, such as PID feedback control, model predictive control, etc.
     - After the baseline feedback control is designed by using the conventional feedback control techniques, the DO-based compensation is added to improve the robustness and disturbance attenuation abilities
   - Less conservativeness:
     - Most of the existing robust control methods are worst-case-based design, where promising robustness is achieved with the cost of the degraded nominal performance, and thus have been criticized as being over-conservative.
     - The nominal performance of the baseline controller is recovered in the absence of disturbances or uncertainties, thus a better nominal dynamic performance would be achieved.

(DOB) 1.3.1 Frequency Domain Formulation



1. Let us consider a minimum-phase single-input, single-output (SISO) linear system

$$
\begin{aligned}
Y(s) &= G_p(s)U(s) + G_p(s)D(s) \\
&= \frac{G_p(s)}{1-Q(s)}[C(s)(Y_r(s)-Y(s)) - Q(s)G_n^{-1}(s)Y(s)] + G_p(s)D(s) \\
G_n(s)Y(s) &= \frac{G_p(s)}{1-Q(s)}[G_n(s)C(s)(Y_r(s)-Y(s)) - Q(s)Y(s)] + G_n(s)G_p(s)D(s) \\
(1-Q(s))G_n(s)Y(s) &= G_p(s)G_n(s)C(s)Y_r(s) - G_p(s)G_n(s)C(s)Y(s) - G_p(s)Q(s)Y(s) + (1-Q(s))G_n(s)G_p(s)D(s)
\end{aligned}
$$

Thus we have

$$
\begin{aligned}
\therefore \quad Y(s) = &\frac{G_p(s)G_n(s)C(s)}{(1-Q(s))G_n(s) + G_p(s)G_n(s)C(s) + G_p(s)Q(s)}Y_r(s) \\
&+ \frac{(1-Q(s))G_p(s)G_n(s)}{(1-Q(s))G_n(s) + G_p(s)G_n(s)C(s) + G_p(s)Q(s)}D(s)
\end{aligned}
$$

2. Here, the disturbance is supposed to be a low-frequency one, which implies that $|D(j\omega)|$ is bounded for a low frequency region $0 < \omega < \omega^*$. For this reason, we can design $Q(s)$ as a *lowpass filter* $Q(j\omega) = 1$ for low frequencies, then we have

$$Y(s) \approx \frac{G_n(s)C(s)}{1 + G_n(s)C(s)} Y_r(s) + 0 \cdot D(s) \quad \text{for low frequencies} \quad \omega \ll \omega^*$$

   where it implies that the real uncertain closed-loop system under the frequency domain DOBC behaves as if it is the nominal closed-loop system in the absence of disturbance, which is also referred to as the *nominal performance recovery*.

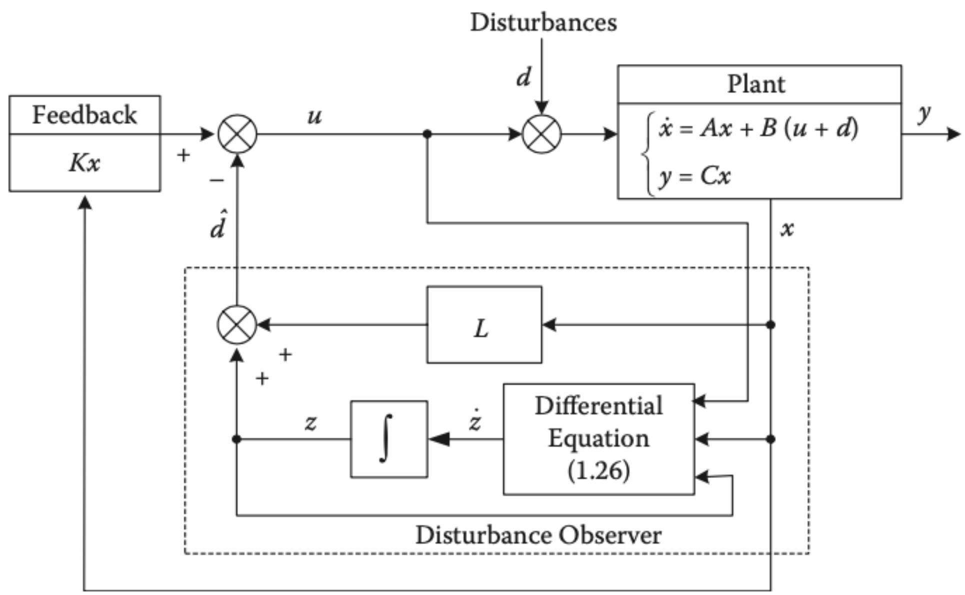
3. On the other hand, for high frequencies, $Q(j\omega) = 0$,

$$Y(s) \approx \frac{G_p(s)C(s)}{1 + G_p(s)C(s)} Y_r(s) + \frac{G_p(s)}{1 + G_p(s)C(s)} D(s) \quad \text{for high frequencies} \quad \omega \gg \omega^*$$

   where it acts like an *actual plant* without Q-filter for high frequencies.

4. Read TAC paper "On the robustness and performance of disturbance observers for second-order systems"

(DOB) 1.3.2 Time Domain DOBC Formulation



1. Consider a multi-input multi-output (MIMO) linear system with disturbances,

$$\dot{x} = Ax + Bu + Bd$$
$$y = Cx$$

2. Suppose that the disturbances and their derivatives are *bounded* and tend to some constants as time goes to infinity, that is,

$$\lim_{t\to\infty} \dot{d}(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} d(t) = d_s$$

where d_s is a constant vector.

3. The following *time domain disturbance observer* can be employed to estimate the disturbances in system

$$\begin{aligned}\dot{z} &= -LB(z + Lx) - L(Ax + Bu)\\ \hat{d} &= z + Lx\end{aligned}$$

where $\hat{d}$ the *disturbance estimation* vector, z the internal variable vector of the observer, and L the observe gain matrix to be designed.

4. Under the time domain framework of DOBC, the control law is generally designed as

$$u = Kx - \hat{d}$$

where K is the feedback control gain to be designed.

5. The *disturbance estimation error* is defined as $e_d = \hat{d} - d$

6. The closed-loop system is governed by

$$\begin{aligned}\dot{x} &= Ax + Bu + Bd = Ax + B(Kx - \hat{d}) + Bd = (A + BK)x - Be_d\\ \dot{e}_d &= \dot{\hat{d}} - \dot{d} = \dot{z} + L\dot{x} - \dot{d} = -LB(z + Lx) - L(Ax + Bu) + L\dot{x} - \dot{d} = -LB\hat{d} + LBd - \dot{d}\\ &= -LBe_d - \dot{d}\end{aligned}$$

where the closed-loop system is asymptotically stable with appropriately chosen parameters K and L, such that $A + BK$ and $-LB$ are Hurwitz.