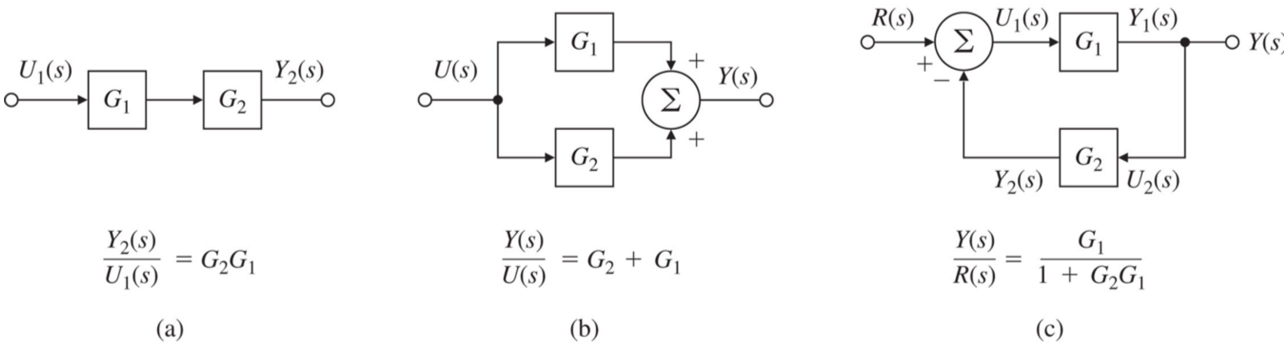
2 System Modeling Diagrams

- Graphical simplification using the TF is easier and more informative than algebraic manipulation
- See the figure 3.9 for series, parallel, and feedback manipulations



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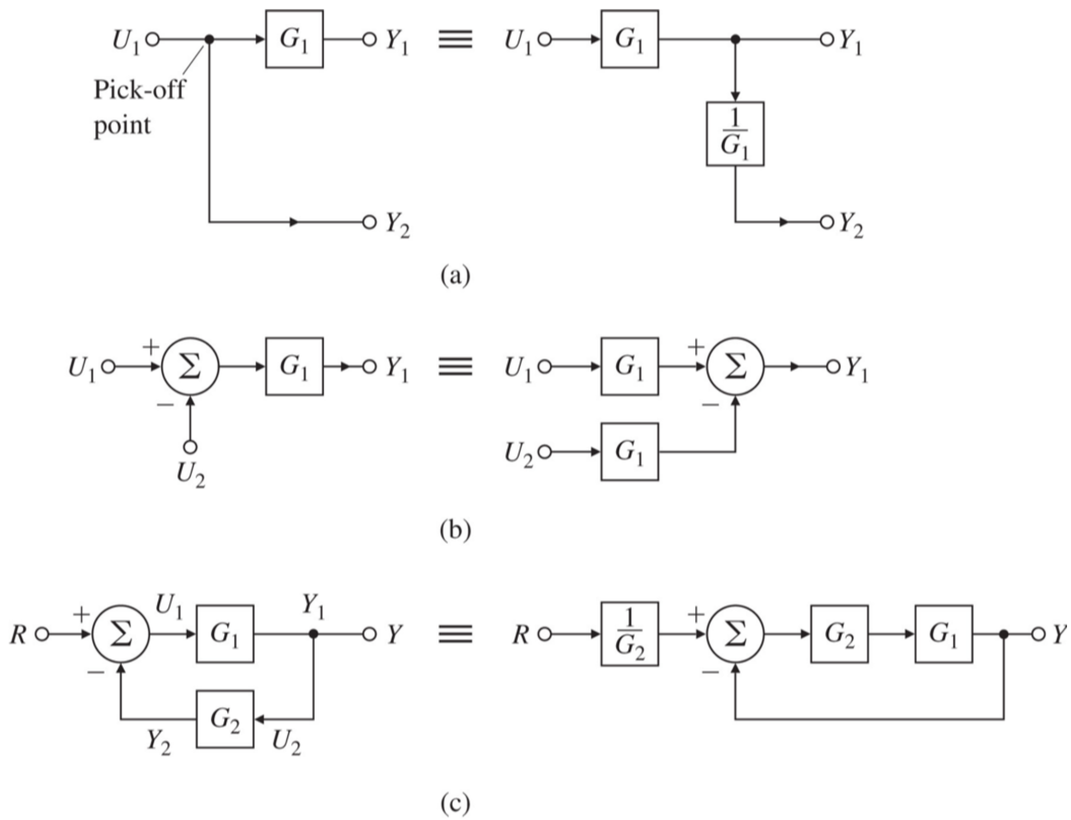
- The gain of a single-loop negative feedback system is given by the forward gain divided by the sum of 1 plus the loop gain

$$Y(s) = \frac{G_1(s)}{1 + G_2(s)G_1(s)} R(s)$$

- See the figure 3.10 for conversions



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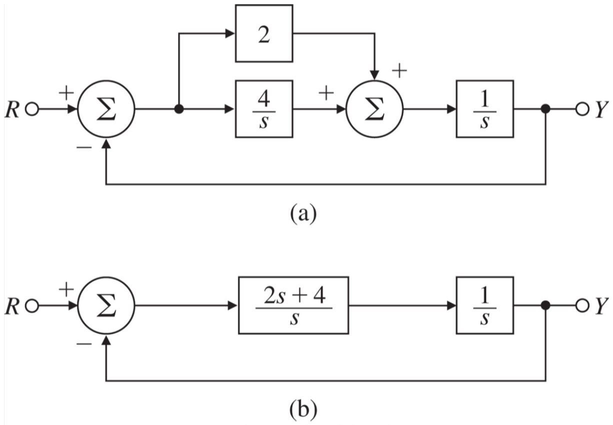
- A system without a component in the feedback path is referred to as a unity feedback system

- (Example 3.22, TF from a Simple Block Diagram) Find the TF of the system shown in Fig. 3.11



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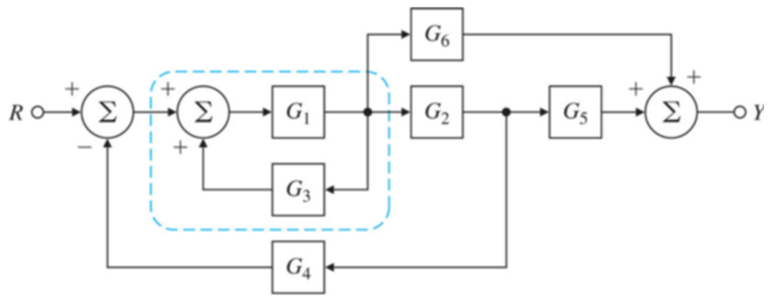
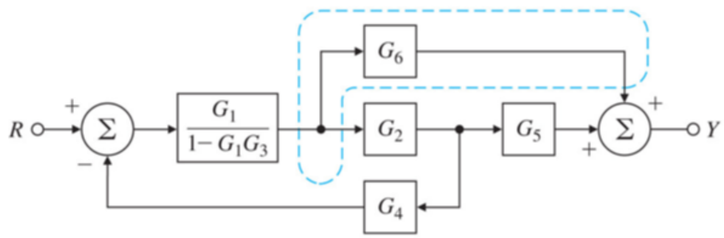
$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{1}{s}(2+\frac{4}{s})}{1+\frac{1}{s}(2+\frac{4}{s})} = \frac{2s+4}{s^2+2s+4}$$

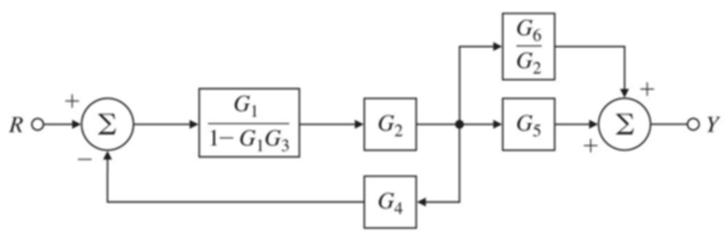
• (Example 3.23, TF from the Block Diagram) Find the TF of the system shown in Fig. 3.12



(a)



(b)



(c)

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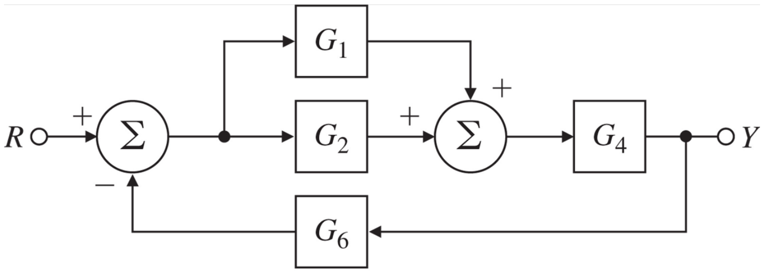
- (Example 3.24, TF of a Simple System using Matlab Simulink) Find the TF of the system shown in Fig. 3.13 with $G_1(s) = 2$, $G_2(s) = \frac{4}{s}$, $G_4(s) = \frac{1}{s}$ and $G_6(s) = 1$



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- Mason's rule is useful technique for determining TFs of complicated interconnected systems. (out of the scope of the textbook, but you can get materials in Appendix W3.2.3 online at www.pearsonglobaleditions.com)

3 Effect of Pole Locations

1. Real Poles

   - Consider the following simple TF

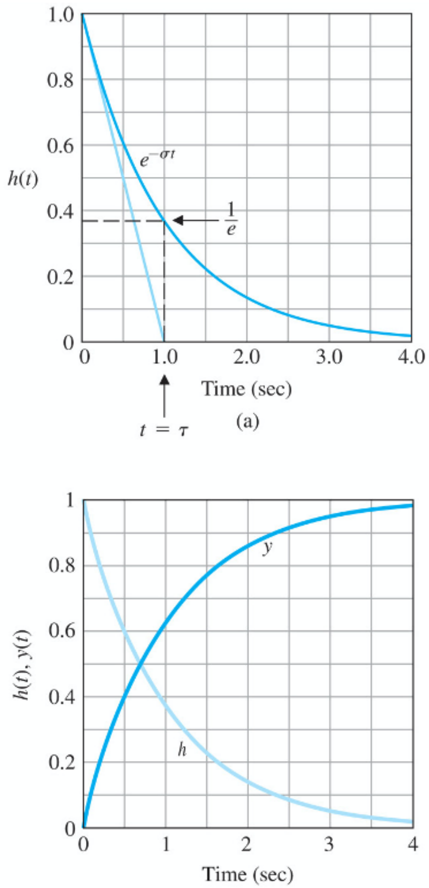
     $$H(s) = \frac{1}{s+\sigma}$$

     where the pole is located at $s = -\sigma$ since it is the point where $H(s)$ is infinity.

   - The impulse response of the TF is

     $$h(t) = e^{-\sigma t} \qquad \text{for } t \geq 0$$

   - When $\sigma > 0$, the pole is located at $s < 0$, the exponential expression decays and we say the impulse response is "stable"
   - If $\sigma < 0$, the pole is to the right of the origin. Because the exponential expression here grows with time, the impulse response is referred to as "unstable".



(a)

(b)

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- Let us introduce the time constant

$$\tau = \frac{1}{\sigma} \qquad \leftarrow \qquad h(t) = e^{-\sigma t} = e^{-\frac{t}{\tau}} \quad \text{for} \quad t \geq 0$$

- The time constant is a measure of the rate of decay (or a measure for the speed of response of the system). The straight line is tangent to the exponential curve at $t = 0$ and terminates $t = \tau$. For example, 63% at $t = \tau$, 86% at $t = 2\tau$, 95% at $t = 3\tau$, 98% at $t = 4\tau$, and 99% at $t = 5\tau$.

- (Example 3.25, Real Roots) Discuss about the impulse response of the following system:

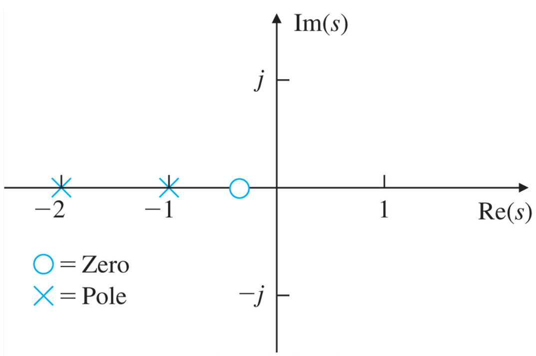
$$H(s) = \frac{2s+1}{s^2+3s+2}$$

a) poles: $s=-1$ and $s=-2$ since they are points that $H(s)=\infty$
b) zeros: $s=-0.5$ and $s=\infty$ since they are points that $H(s)=0$
c) partial fraction expression:

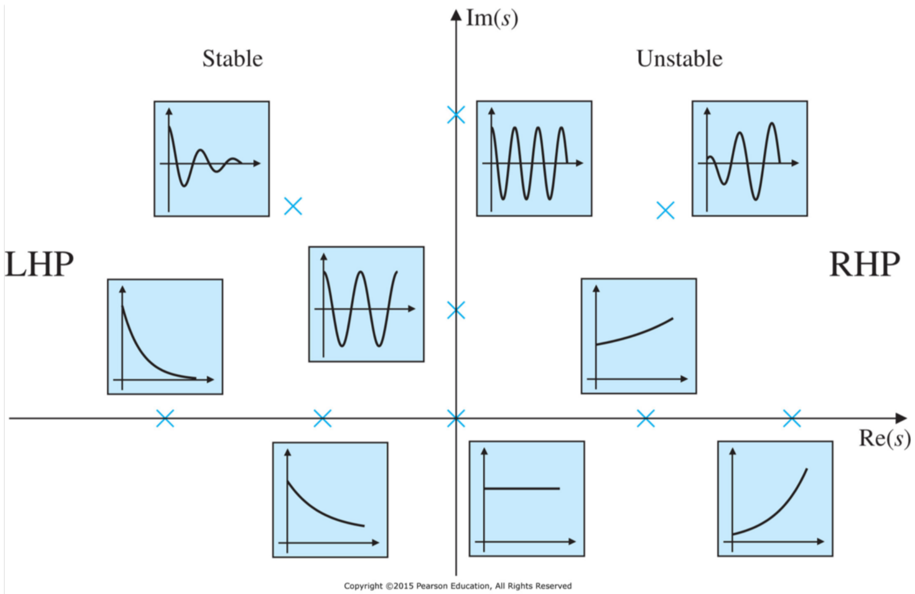
$$H(s) = \frac{2s+1}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{3}{s+2}$$

d) impulse response:

$$h(t) = 3e^{-2t} - e^{-t} \qquad \text{for } t \geq 0$$



A sketch of these pole locations and corresponding natural responses is given in Fig. 3.16, along with other pole locations including complex ones.



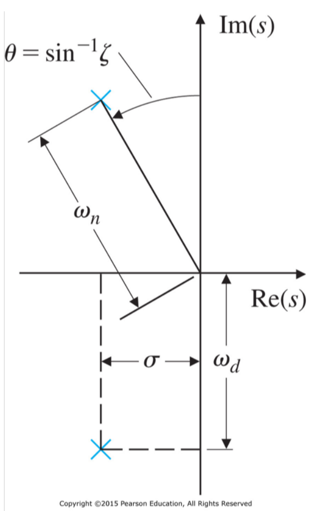
a) Poles farther to the left in the s-plane are associated with natural signals that decay faster than those associated with poles closer to the imaginary axis.
b) In the response of $h(t) = 3e^{-2t} - e^{-t}$, the fast $3e^{-2t}$ term dominates the early part of the time history and the $-e^{-t}$ term is the primary contributor later on.

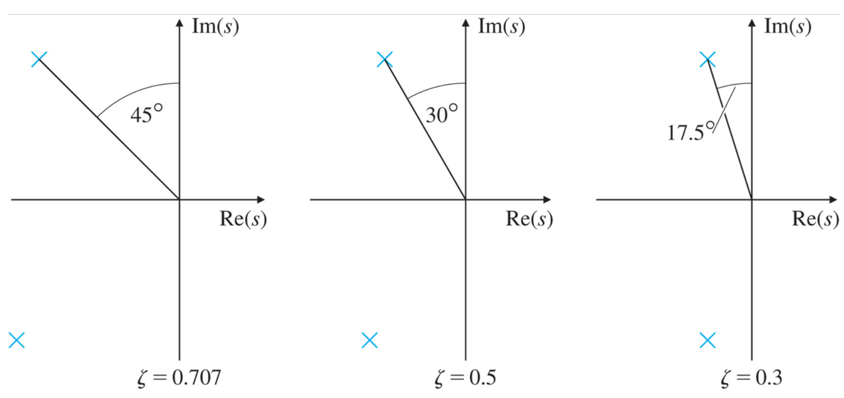
2. Complex Poles

- Consider the typical second-order system with $0 < \zeta < 1$:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

a) complex poles : $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$
b) damping ratio : ζ
c) Neper frequency : $\sigma = \zeta\omega_n$
d) undamped natural frequency : ω_n
e) damped natural frequency : $\omega_d = \omega_n\sqrt{1-\zeta^2}$
f) The poles of this TF are located at a radius ω_n in the s-plane and at an angle $\theta = \sin^{-1}\zeta$, as shown in Figs. 3.18 and 3.20



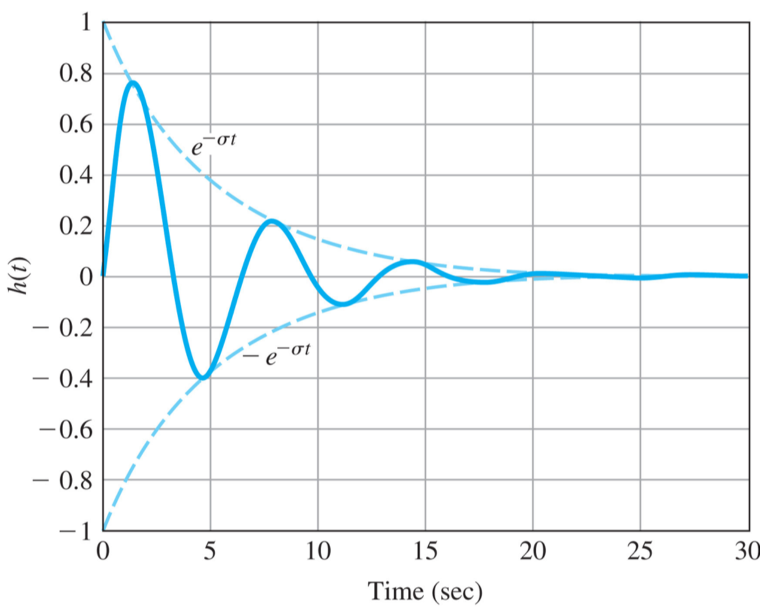


- Impulse response becomes

$$
\begin{aligned}
H(s) &= \frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} \\
&= \frac{\omega_n^2}{\omega_d}\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2} \\
&= \frac{\omega_n}{\sqrt{1-\zeta^2}}\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2} \qquad \to \qquad \mathcal{L}[e^{-\sigma t}f(t)] = F(s+\sigma) \\
h(t) &= \frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t \qquad \textbf{for} \quad t \geq 0
\end{aligned}
$$

a) Actual frequency ω_d decreases slightly as the damping ratio increases

b) Negative real part of the pole $\sigma = \zeta\omega_n$ determines the decay rate of an exponential envelope ($\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}$ and $-\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}$) that multiplies the sinusoid as shown in Fig. 3.21



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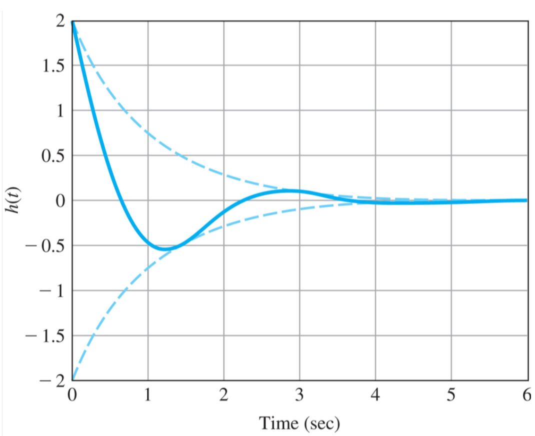
- Stability
  a) If $\sigma < 0$ (and the pole is in the RHP), then the natural response will grow with time, so, as defined earlier, the system is said to be unstable.
  b) If $\sigma = 0$, the natural response neither grows nor decays, so stability is open to debate.
  c) If $\sigma > 0$, the natural response decays, so the system is stable.

- (Example 3.26, Oscillatory Time Response) Discuss the correlation b/w the poles and impulse response of the system:

$$
\begin{aligned}
H(s) &= \frac{2s+1}{s^2+2s+5} = \frac{2s+1}{(s+1)^2+2^2} \\
&= 2\frac{s+1}{(s+1)^2+2^2} - \frac{1}{2}\frac{2}{(s+1)^2+2^2} \\
h(t) &= 2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t \qquad \text{for } t \geq 0 \\
&= \frac{\sqrt{17}}{2}e^{-t}\cos\left(2t + \tan^{-1}\frac{1}{4}\right)
\end{aligned}
$$

a) $\omega_n = \sqrt{5} = 2.24$ **from** $\omega_n^2 = 5$
b) $\zeta = \frac{1}{\sqrt{5}} = 0.447$ **from** $2\zeta\omega_n = 2$
c) $\omega_d = 2$ **from** $\omega_d = \omega_n\sqrt{1-\zeta^2}$
d) **both envelopes are** $\frac{\sqrt{17}}{2}e^{-t}$ **and** $-\frac{\sqrt{17}}{2}e^{-t}$
e) **impulse response is plotted in Fig. 3.22**



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