In this paper, an inverse dynamics-based torque tracking control method for rotary-type hydraulic actuators is proposed. The effectiveness of the proposed control method is shown through simulation.

**NOMENCLATURES**

- \( \beta \) : Effective bulk modulus
- \( D \) : Volume displacement
- \( c_p \) : Discharge coefficient of valve
- \( c_l \) : Internal leakage
- \( c_{el} \) : External leakage
- \( V_1, V_2 \) : Trapped fluid volumes in both chambers
- \( V_0 \) : Initial fluid volume
- \( x \) : Angular displacement
- \( \dot{x} \) : Angular velocity
- \( p_1, p_2 \) : Pressure inside two chambers
- \( u \) : Servo-valve displacement (control input)
- \( p_a \) : Supply pressure
- \( \tau \) : Hydraulic torque

\[
\frac{1}{\beta} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \frac{c_l}{V_1(x)} - \frac{c_{el}}{V_1(x)} \\ \frac{c_l}{V_2(x)} - \frac{c_{el}}{V_2(x)} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} c_l \\ c_{el} \end{bmatrix} - \frac{V_1(x)}{V_2(x)} \dot{x} + c_p \begin{bmatrix} \sqrt{K_1(sgn(u), p_1)} \\ -\sqrt{K_2(sgn(u), p_2)} \end{bmatrix} u \tag{1}
\]

where

\[
V_1(x) = V_0 + Dx \\
V_2(x) = V_0 - Dx
\]

\[
K_1(sgn(u), p_1) = 0.5(p_1 - p_a) + sgn(u)[0.5(p_1 + p_a) - p_1] \\
K_2(sgn(u), p_2) = 0.5(p_2 - p_a) - sgn(u)[0.5(p_2 + p_a) - p_2]
\]

in which \( V_1 \) and \( V_2 \) imply the trapped fluid volumes in chamber 1 and 2, respectively, \( K_1 \) and \( K_2 \) express dynamic relationships between the time derivative of pressures in both chambers and the servo-valve displacement.

Also the hydraulic torque is generated proportional to the pressure difference between both chambers as follow:

\[
\tau = D(p_1 - p_2) \tag{2}
\]

Now if we take time derivative of above equation using Eq. (1), then we have

\[
\dot{\tau} = D(p_1 - p_2) = -\beta D(c_l + c_{el})P_+(x)(p_1 - p_2) + \beta Dc_elp_-(x)p_a - \beta D^2P_+(x)\dot{x} + \beta Dc_pQ(p_1, p_2, x, sgn(u))u \tag{3}
\]

where

\[
P_+(x) = \frac{1}{V_1(x)} + \frac{1}{V_2(x)} = \frac{1}{V_0 + Dx} + \frac{1}{V_0 - Dx} \\
P_-(x) = \frac{1}{V_1(x)} - \frac{1}{V_2(x)} = \frac{1}{V_0 + Dx} - \frac{1}{V_0 - Dx} \\
Q(p_1, p_2, x, sgn(u)) = \sqrt{K_1(sgn(u), p_1)}V_1(x) + \sqrt{K_2(sgn(u), p_2)}V_2(x)
\]
Due to structural property of the rotary-type hydraulic actuator shown in the Fig. 1, the angular displacement is normally bounded with upper and lower limits, such as \( x \in (-D^{-1}V_o, D^{-1}V_o) \) and \( \beta Dc_pQ(p_1, p_2, x, u) \neq 0 \). Furthermore, the inner-loop control can be designed for the inverse dynamics-based control method as follow:

\[
u = \frac{1}{\beta Dc_pQ(p_1, p_2, x, \text{sgn}(u^*))} \left\{ \beta D(c_l + c_d)P_+(x)(p_1 - p_2) - \beta Dc_dP_-(x)p_a + \beta D^2P_+(x)\dot{x} + v \right\}
\]  

(4)

where \( v \) is an outer-loop control input to be designed later and

\[\text{sgn}(u^*) = \text{sgn} \left\{ \beta D(c_l + c_d)P_+(x)(p_1 - p_2) - \beta Dc_dP_-(x)p_a + \beta D^2P_+(x)\dot{x} + v \right\}
\]  

(5)

By applying the inner control input of Eq. (4) to Eq. (3), the closed-loop control system is obtained as follow:

\[
\cdot t = v
\]  

(6)

Till now, we have reviewed the dynamics and the inverse dynamics-based inner loop control. Next section will suggest the outer-loop controller for torque tracking task.

II. TORQUE TRACKING CONTROL

For given desired torque profiles \( \tau_d(t) \) and \( \tau_d(t) \), the torque tracking control is proposed as the outer-loop controller:

\[
v = \tau_d + K_p(\tau_d - \tau) + K_i \int (\tau_d - \tau) dt
\]  

(7)

with \( K_p > 0 \) and \( K_i > 0 \) imply proportional and integral gains, respectively. If we apply Eq. (7) to Eq. (6), then we have

\[\dot{\tau} + K_p\tau + K_i \int \tau = 0\]

where \( \tau \) is a torque error defined as the difference between \( \tau_d \) and \( \tau \). In addition, the asymptotic stability of torque error is easily proven with the conditions of \( K_p > 0 \) and \( K_i > 0 \). In other words, \( \tau(t) \rightarrow \tau_d \) as \( t \rightarrow \infty \).

III. SIMULATION AND CONCLUDING REMARKS

For simulation study, assume that a rigid link is attached to the rotation axis as shown in Fig. 2, then the link dynamics can be obtained including gravitational torque and friction effect as follow:

\[ml^2\ddot{x} + \mu \cdot \text{sgn}(x) + mgl\sin(x) = \tau + \tau_{\text{ext}}\]  

(8)

where \( \tau_{\text{ext}} \) implies the external torque. Assume that the physical parameters for the hydraulic actuator are given as \( c_l = c_d = 1.11 \times 10^{-7}, c_p = 1.49 \times 10^{-4}, V_0 = 1.67 \times 10^{-4}, D = 2.66 \times 10^{-5}, p_a = 10^5, p_s = 20, 684, 280, \) and \( \beta = 14, 400 \). Also assume that the link parameters are given as \( m = 1[kg], l = 1[m], g = 10[m/s^2], \mu = 0.5, \) and \( \tau_{\text{ext}} = 0 \). Furthermore, the initial conditions are given as \( x(0) = 0, \dot{x}(0) = 0, p_1(0) = p_s/2, p_2(0) = p_s/2, u(0) = 0 \). The simulation procedures from \( t = 0 \) are summarized as follows:

1) Determine \( \tau_d(t) = 3\sin(2\pi t) \) and \( \tau_d(t) = 6\pi\cos(2\pi t) \)
2) Find \( v \) from Eq. (7)
3) Find \( \text{sgn}(u^*) \) from Eq. (5)
4) Find \( u \) from Eq. (4)
5) Determine \( p_1(t) \) and \( p_2(t) \) by solving ODE (1)
6) Find \( \tau \) from Eq. (2)
7) Determine \( x \) and \( \dot{x} \) by solving ODE (8)
8) \( t = t + \Delta T, \) then go to step 1

The simulation results are shown in Fig. 3. Here we can confirm from the Fig. 3 that the proposed torque tracking control showed a good performance while reducing the torque error.

![Fig. 2. Rotary-type Hydraulic Actuator with Link](Image 104x118 to 232x248)

Fig. 2. Rotary-type Hydraulic Actuator with Link

![Fig. 3. Simulation Results](Image 319x242 to 535x407)

Fig. 3. Simulation Results

Based on the dynamics of rotary-type hydraulic actuator presented in [1], both the outer-loop and inner-loop controller were proposed for the inverse dynamics-based control. Finally, the effectiveness of the proposed control method was shown through the simulation.

REFERENCES