1 Introduction

The PID controller for Lagrangian systems has been widely used with various usages. Also, it has reproduced a great variety of PID control plus something, e.g., PID plus friction compensator, PID plus gravity compensator, PID plus disturbance observer, etc. As a matter of fact, the importance of PID control comes from the easy applicability and clear effects of each proportional, integral and derivative control. Inspired by the extended disturbance input-to-state stability of PID trajectory tracking control for Lagrangian systems, the inverse optimality of PID controller was proved in [1,2] with some conditions for gains. Also, it was proved that the inverse optimal PID controller brings the extended disturbance input-to-state stability of Lagrangian systems. Recently, the noticeable “square tuning” and “linear tuning” rules were proposed using an inverse optimal PID controller. They were derived from the performance prediction equation suggested in [2,3]. In fact, the performance tuning by the name of square law had been suggested for the first time in [4]. Also, the compound tuning rule unifying both square tuning and linear tuning will be newly proposed in this paper. However, this tuning method is passive in that the controller can be adjusted after we confirm the performance of applied controller.

Many automatic performance tuning methods of PID controller were proposed in [5–7], however, they are for the chemical process control systems. Since process systems show very slow responses with time-delay effect, the autotuning algorithms developed for process control systems cannot be directly applied to Lagrangian systems. Though the performance tuning by gain changes has brought one’s interest with wide acceptance, there still exist no generally applicable autotuning laws of PID controller for Lagrangian systems. In this paper, an automatic performance tuning method of an inverse optimal PID trajectory tracking controller will be proposed by making use of the direct adaptive control scheme based on the extended disturbance input-to-state stability and the analysis result of performance limitation. Recently, the direct adaptive control scheme for nonlinear systems was developed in [8–11]. Especially in [8], Haddad and Hayakawa suggested the direct adaptive control method with $L_2$-gain disturbance attenuation for nonlinear systems. The direct adaptive control is different from the indirect adaptive control in that the control parameters are estimated directly without intermediate calculations involving plant parameter estimates. Strictly speaking, the conventional adaptive motion (trajectory tracking) control methods given in [12–14] can be classified as the indirect adaptive control for Lagrangian system, especially a robotic manipulator, because the parameters of Lagrangian system are estimated to construct a dynamical model compensator. As far as we know, the direct adaptive control for nonlinear dynamical system similar to Lagrangian system was investigated for the first time in [11]. In a viewpoint of direct adaptive control, an autotuning law of PID trajectory tracking controller will be proposed in this paper.

The notion of disturbance input-to-state stability (ISS) is helpful to understand the effect of disturbances acting on system states. When there exist unknown bounded disturbances such as perturbations and external disturbances, the behavior of system should remain bounded. These characteristics of ISS were investigated thoroughly in [15–18]. Also, the control system is said to be disturbance input-to-state stable (ISS) if there exist a class $K_C$ function $\beta$ and a class $K$ function $\gamma$ such that the solution trajectory for a given system exists for all $t \geq 0$ and satisfies

$$\|x(t)\| \leq \beta(\|x(0)\|, t) + \gamma(\|w\|_\infty),$$  

for an initial state vector $x(0)$ and for a disturbance vector $w(\cdot)$ piecewise continuous and bounded on $[0, \infty)$. Here, the Euclidian norm and $L_\infty$ norm of $x(t)$ at time $t$ are defined in the following forms:

$$\|x(t)\| = \sqrt{x(t)'x(t)}$$

$$\|x\|_\infty = \sup_{0 \leq t \leq T} |x(t)|.$$  

Also, there exists a smooth positive definite radially unbounded function $V(x, t)$, a class $K_C$ function $\gamma_1$ and a class $K$ function $\gamma_2$ such that the following dissipativity inequality is satisfied:

$$\dot{V} \leq -\gamma_1(\|x\|) + \gamma_2(\|w\|),$$  

if and only if the system is ISS, where $\dot{V}$ represents the total derivative of the Lyapunov function.

This paper is organized as follows: the performance limitation of PID trajectory tracking controller and its passive performance tuning method will be suggested in Section 2. In Section 3, an automatic performance tuning law of PID controller will be de-
2 Performance Limitation and Tuning

In this section, we investigate the performance limitation imposed by applying an inverse optimal PID controller to Lagrangian systems. The design method of an inverse optimal PID controller was suggested in [1,2]. First of all, the trajectory tracking system model will be obtained for Lagrangian systems.

2.1 Trajectory Tracking System Model. Lagrangian system is described by using Lagrangian equation of motion with configuration coordinates \( q=[q_1, q_2, ..., q_n]^T \) as follows:

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + d(t) = \tau,
\]

where \( M(q) = \dot{M}(q) \in \mathbb{R}^{n \times n} \) is inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^n \) Coriolis and centrifugal torque vector, \( g(q) \in \mathbb{R}^n \) gravitational torque vector, \( \tau \in \mathbb{R}^n \) control input torque vector and \( d(t) \) unknown external disturbances. Most mechanical systems can be therefore classified as Lagrangian systems because they can be described by the Lagrangian equation of motion. In fact, Lagrangian system (3) is also the system model for set-point regulation control. Here, let us introduce the extended disturbance defined in the following form:

\[
w(t, \dot{e}, \int e) = M(q)[\dot{q}_d + K_p \dot{e} + K_p e] + C(q, \dot{q}) \left( \dot{q}_d + K_p \dot{e} + K_p e \right) + g(q) + d(t),
\]

where \( K_p, K_i \) are positive constant diagonal matrices, \( e=q_q-q \) is the configuration error vector and desired configurations \( \dot{q}_d, \dot{q}, \dot{q}_d \) are the function of time, hence, the extended disturbance \( w \) is the function of time, configuration error, its derivative and integral, because \( q=q_d-e \) and \( \dot{q}=(\dot{q}_d-e) \) are the function of time, configuration error, and its derivative. If the extended disturbance defined above is applied to the set-point regulation system model (3), then the trajectory tracking system model can be obtained as

\[
M(q)\ddot{s} + C(q, \dot{q})\dot{s} = w(t, \dot{e}, \int e) + u,
\]

where the control input and composite error vector are defined as follows:

\[
u = -\tau
\]

\[
s = \dot{e} + K_p \dot{e} + K_i \int \dot{e} dt.
\]

The difference and common point between the set-point regulation system model (3) and trajectory tracking system model (5) are explained in following Remark.

Remark 1: If the trajectory tracking system model (5) can be stabilized by some controller, then Lagrangian system (3) is stabilized by same controller because the boundedness of \( s \) implies those of \( q \) and \( \dot{q} \). Also, if the set-point regulation system model (3) is rewritten by using state vector \( q \) as follows:

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} = w_r(t, q) + \tau,
\]

where \( w_r(t, q) = -g(q) - d(t) \), then we can see that the trajectory tracking system (5) and set-point regulation system (6) show the same dynamic characteristics such as \( M(q)^{-1}C(q, \dot{q}) \).

2.2 Performance Limitation. In this section, we are to obtain the performance limitation imposed by applying an inverse optimal PID controller suggested in [1–3] to trajectory tracking system model. Here, an inverse optimal PID controller has the following form:

\[
\tau = \left( K + \frac{1}{\gamma} I \right) \left( \dot{e} + K_p \dot{e} + K_i \int \dot{e} dt \right),
\]

and its design conditions are as follows:

(C1) \( K, K_p, K_i > 0 \), constant diagonal matrices
(C2) \( K_p > 2 K_i \).

From now on, we will omit terms “inverse optimal” since the utilized PID controllers will have the form of an inverse optimal PID controller of (7). Also, if above PID controller is applied to the trajectory tracking system (5), then the state vector of closed-loop system is upper bounded by the extended disturbance as suggested in following Theorem. As a matter of fact, the bound of composite error in following Theorem means the performance limitation of PID controller for trajectory tracking system.

Theorem 1: Let \( s = \dot{e} + K_p \dot{e} + K_i \int \dot{e} dt \in \mathbb{R}^n \). If the PID controller (7) is applied to the trajectory tracking system (5), then the composite error is upper bounded in the following form:

\[
|s(t)| \leq |s(0)| e^{-[(2k^2 + 0.5\gamma^2)/\lambda]t} + \frac{\gamma^2}{2k^2 + 1} |w|^2.
\]

where \( w(0) \) is the initial composite error vector, \( \lambda \) is a maximum eigenvalue of inertia matrix \( M \), and \( k \) is a minimum diagonal element of \( K \).

Proof: If the PID controller of (7) is applied to (5), then the closed-loop control system becomes as follows:

\[
M\ddot{s} + C\dot{s} = w - \left( K + \frac{1}{\gamma} I \right) s.
\]

Since above closed-loop system is also Lagrangian system, the characteristics of Lagrangian system can be used for above system. An useful characteristics is that the equality \( M = C + C^T \) is always satisfied. Now, let us differentiate the positive real-valued function \( s^2 / 2 \) along (9) as follows:

\[
\frac{d}{dt} \left( \frac{1}{2} s^2 M s \right) = \frac{1}{2} s^2 M s + \frac{1}{2} s^2 M s + \frac{1}{2} s^2 M s = s^T \left( K + \frac{1}{\gamma} I \right) s + 2 s^T w,
\]

by using \( M = C + C^T \)

\[
= -s^T \left( K + \frac{1}{\gamma} I \right) s - \gamma^2 |w|^2, \quad \text{by the above condition,}
\]

\[
= -s^T \left( K + \frac{1}{\gamma} I \right) s + \frac{\gamma^2}{2} |w|^2.
\]

Using the maximum eigenvalue of Inertia matrix \( M \) and the minimum diagonal element of gain matrix \( K \), above inequality can be simplified to:

\[
\frac{d}{dt} \left( \frac{1}{2} s^2 \right) \leq \left( k + \frac{1}{\gamma} \right) |s|^2 + \frac{\gamma^2}{2} |w|^2.
\]

If we multiply above inequality by \( e^{[(2k^2 + 0.5\gamma^2)/\lambda]t} \), then it becomes

\[
\frac{d}{dt} \left( \lambda |s|^2 e^{[(2k^2 + 0.5\gamma^2)/\lambda]t} \right) \leq \gamma^2 |w|^2 e^{[(2k^2 + 0.5\gamma^2)/\lambda]t}.
\]

Integrating (10) over \([0, t]\), we arrive at the following form:
\[ |s(t)|^2 = |s(0)|^2 e^{-\frac{(2k)^2}{\lambda} \tau} + \frac{\gamma^2}{\lambda} \int_0^\tau e^{-\frac{(2k)^2}{\lambda} \tau} \|w(\tau)\|^2 d\tau \]

\[ = |s(0)|^2 e^{-\frac{(2k)^2}{\lambda} \tau} + \frac{\gamma^2}{\lambda} \sup_{\tau \in [0,t]} \|w(\tau)\|^2 e^{-\frac{(2k)^2}{\lambda} \tau} \]

\[ \times \int_0^\tau e^{-\frac{(2k)^2}{\lambda} \tau} \|w(\tau)\|^2 e^{-\frac{(2k)^2}{\lambda} \tau} d\tau \]

\[ = |s(0)|^2 e^{-\frac{(2k)^2}{\lambda} \tau} + \frac{\gamma^2}{\lambda} \left( \|w(\tau)\|^2 e^{-\frac{(2k)^2}{\lambda} \tau} + 1 \right). \]

By applying the property of \(|a + b|^2 \leq |a|^2 + |b|^2\) to right-hand side of the above inequality, an explicit upper bound of composite error vector is obtained as follows:

\[ |s(t)| = |s(0)| e^{-\frac{k^2 y^2}{\lambda} t} + \frac{\gamma^2}{\lambda} \sup_{t \in [0,T]} \|w(t)\| \sqrt{1 - e^{-\frac{2k^2 y^2}{\lambda} t}} \]

\[ \leq |s(0)| e^{-\frac{k^2 y^2}{\lambda} t} + \frac{\gamma^2}{\lambda} \|w(\tau)\|. \]

The first term of the right-hand side of (8) is a \(KL\) function because it is an increasing function for \(|s(0)|\) and decreasing one for \( t \). Also, the second term is a \( KL\) function since it is an increasing one for \( \|w(\tau)\|\). Hence, the extended disturbance input-to-state stability (ISS) can be also proved from Theorem 1 because the upper bound (8) follows the ISS characteristics of (1).

Though the exponential term of (8) goes to zero as \( t \to \infty \), the composite error can not be zero because the extended disturbance of (4) includes the inverse dynamics according to desired configurations \((q_d, \dot{q}_d, \ddot{q}_d)\) and gravitational torque \(g(q)\), moreover, \( w \neq 0 \) as shown in the following equation even when \( e=0, \dot{e}=0, \ddot{e}=0\) and \( edt = 0\):

\[ w(t, e, e, edt) = Mq + Cq + g + d + MKe + (MK + CK)e + CK, \int edt. \]

Also, the size of the composite error is affected by the size of the extended disturbance as shown in Eq. (8). As a matter of fact, the boundedness of (8) expresses the performance limitation imposed by using a PID trajectory tracking controller. Although the PID trajectory tracking controller guarantees the extended disturbance input-to-state stability (ISS) for Lagrangian systems, it does not bring the globally asymptotic stability (GAS). This was proved for the first time in [2]. Also, the upper bound of composite error naturally suggests new performance tuning rule.

2.3 Passive Performance Tuning. The performance tuning method can be perceived from the relation between gain and error. In the trajectory tracking control, since the initial composite error \( s(0) \) of (8) can be set to zero vector by the initialization of control system, the composite error can be bounded only by \( L_\infty \) norm of the extended disturbance as follows:

\[ |s(t)| \leq \frac{\gamma^2}{\sqrt{2k^2 Y^2} + 1} \|w\|. \]  

As a matter of fact, the boundedness of (12) includes the performance tuning law representing the relation between the composite error \( s \) and gains \((k, \gamma)\) of the PID controller. Then, the extended disturbance shows almost same magnitude for same trajectory because it is largely affected by inverse dynamics according to desired configurations. Therefore, if the utilized PID controller can stabilize the system, then we can find the following proportional relation from (12):

\[ |s| \approx \frac{\gamma^2}{\sqrt{2k^2 Y^2} + 1}. \]  

Above is the “compound tuning rule” unifying both square and linear tuning rules suggested in [2]. Also, this is a passive performance tuning method in that above tuning rule can be applied after we performed the experiment once at least.

Remark 2: In [2], the square and linear tuning rules were proposed and proved through the experiments. For a composite error, these square and linear tuning rules can be also found by approximating (13) according to the size of gain \( k \) as follows:

- **Square Tuning:** \(|s| \approx \gamma^2\), for a small \( k\).
- **Linear Tuning:** \(|s| \approx \gamma^2\), for a large \( k\).

Although above rules are very useful in tuning the control performance, they can be utilized only by repetitive experiments for same trajectory because the tuning rules consist of proportional relations. Hence, this also correspond to passive performance tuning methods.

2.4 Numerical Example: Compound Tuning Rule. Let us consider the simple pendulum system as shown in Fig. 1, then its equation of motion is described as follows:

\[ ml^2 \ddot{q} + mgl \sin(q) = \tau, \]

and the trajectory tracking system model is obtained as follows:

\[ ml^2 \ddot{s} = w(t, e, \ddot{e}) + u, \]

by defining the extended disturbance and composite error in the following forms:

\[ w(t, e, \ddot{e}) = ml^2 (\ddot{q}_d + k \dot{e} + k_\dot{e}) + mgl \sin(q), \]

\[ s = \dot{e} + k_\dot{e} + k_1 \int e dt \]

\[ u = -\tau. \]

Also, if the PID controller as following form is applied to above pendulum system:

\[ \tau = (k + \frac{1}{\gamma})(\dot{e} + k_\dot{e} + k_1 \int e dt), \]

then the closed-loop control system can be obtained as follows:

\[ ml^2 \ddot{s} + (k + \gamma^2) s = w(t, e, \ddot{e}). \]

To show the validity of compound tuning rule (13), first, the desired trajectory is determined as the fifth order polynomial function as shown in Fig. 2. Second, the plant parameters and gains are determined like this: \( m=1 \text{ kg}, l=1 \text{ m}, g=9.806 \text{ m/s}^2, k_P=20, k_I=100 \). Finally, the simulation results were obtained as shown in Fig. 3 according to the change of \( \gamma \) when \( k=10 \). In that figure, we can see that the control performances comply with the compound performance tuning rule (13). For instance, the maximum values...
of composite errors in Fig. 3(a) were rearranged in Table 1. Here, the proportional constant determined by gains of (13) are calculated as 0.0781 for $\gamma=0.4$ and 0.0298 for $\gamma=0.2$. Therefore, the performance will be enhanced by 2.621 times since (0.0781 $\div$ 0.0298), in other words, the composite error will be reduced by 1/2.621 times. As we can see in Table 1, the real composite errors comply well with the compound tuning rule. The maximum deviation between the real performance enhancement and expected one is 5.1% in Table 1. Since the configuration error has the proportional relation with the composite error, the configuration errors comply also well with the compound tuning rule as shown in Fig. 3(b) and Table 2. The maximum deviation is 5.9% in Table 2.

As a matter of fact, the advantage of compound tuning rule is that it unifies both the square tuning rule and linear one explained in Remark 2 as one tuning rule. However, since the compound tuning rule can be applied after we implemented the experiment once at least, this is also the passive performance tuning method. To make active use of the passive tuning rule, we will devise the automatic performance tuning method in following section.

## 3 Automatic Performance Tuning

In this section, the automatic performance tuning method of PID controller for Lagrangian systems will be proposed using the concept of direct adaptive control. Since the PID controller shows the performance limitation for trajectory tracking model of Lagrangian system as shown in previous section, the automatic performance tuning method is devised so that it can accomplish the

---

### Table 1

| $\gamma$ | $||s||_\infty$ | Expected, $\left(\frac{\gamma^2}{\sqrt{2k\gamma^2+1}}\right)$ | $\frac{||s||_\infty}{||s||_{\infty,\gamma}}$ |
|---|---|---|---|
| 0.4 | 0.7720 | 2.495 | 2.621 |
| 0.2 | 0.3094 | 3.230 | 3.275 |
| 0.1 | 0.0958 | 3.742 | 3.792 |
| 0.05 | 0.0256 | | |

### Table 2

| $\gamma$ | $||s||_\infty$ | Expected, $\left(\frac{\gamma^2}{\sqrt{2k\gamma^2+1}}\right)$ | $\frac{||s||_\infty}{||s||_{\infty,\gamma}}$ |
|---|---|---|---|
| 0.4 | 0.0250 | 2.475 | 2.621 |
| 0.2 | 0.0101 | 3.483 | 3.275 |
| 0.1 | 0.0029 | 3.625 | 3.792 |
| 0.05 | 0.0008 | | |
target performance of PID control system. The target performance implies that the composite error should stay within ball boundary \((\Omega)\) chosen by the user, in other words, \(|x(t)| \leq \Omega\) for all \(t\). To begin with, let us obtain the state-space description for trajectory tracking system model in following section.

### 3.1 State-Space Description

First, if we define \(3n\)-dimensional state vector as follows:

\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} e(t) \\ e \\ e \end{bmatrix} \in \mathbb{R}^{3n},
\]

then the state-space representation of trajectory tracking system model (5) can be obtained as follows:

\[
x(t) = A(x,t)x + B(x,t)w + B(x,t)u
\]

where

\[
A(x,t) = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ -M^{-1}CK_x - M^{-1}CK_p - K_t - M^{-1}C - K_p \end{bmatrix}
\]

and

\[
B(x,t) = \begin{bmatrix} 0 \\ 0 \\ M^{-1} \end{bmatrix}
\]

This state-space description for trajectory tracking system model is one of generic forms suggested in [19].

Second, let us investigate the characteristics of extended disturbance as explained in previous section, we will define the quasi-equilibrium region and find it for above closed-loop system in the following theorem.

**Theorem 2:** If the quasi-equilibrium region is defined as the interior region of ball with the largest radius among state vectors satisfying \(x = 0\), then it is obtained in the following form:

\[
|x| \leq \|x_\infty\| = \sup_{0 \leq t \leq \ell} |x(t)|,
\]

where \(x_\infty(t)\) means the state vector satisfying \(x = 0\) in (18) and its Euclidian norm is as follows:

\[
\|x_\infty\| = \frac{\| \int_0^\ell \dd x(t) \|}{\ell}.
\]

Since above closed-loop system has no equilibrium points as explained in previous section, we will define the quasi-equilibrium region as follows:

\[
x_\infty(t) = K_y^{3}[K + \gamma^{-2}I]Y_y(t)\theta,
\]

where

\[
\dot{x} = A_x(x,t)x + B_x(x,t)Y_y(x,t)\theta
\]

and

\[
A_x = A - B(K + \gamma^{-2}I)[K_y, K_p, I].
\]

Also, \(Y_y(t)\) and \(Y_y(x,t)\) have the following forms:

\[
B(x,t) = \begin{bmatrix} 0 \\ 0 \\ M^{-1} \end{bmatrix},
\]

\[
Y_y(x,t) = M(q)\dot{q}_d + C(q,q)\left(\dot{q}_d + K_p\dot{q} + K_t I \right) e + g(q_d).
\]

Finally, the quasi-equilibrium region is obtained by its definition as follows:

\[
|x| \leq \|x_\infty\| = \sup_{0 \leq t \leq \ell} |x(t)|,
\]

where \(x_\infty(t) = K_y^{3}[K + \gamma^{-2}I]Y_y(t)\theta\).

The size of the quasi-equilibrium region is inversely proportional to the integral gain \(K_y\) as we can see in Eq. (19). Also, large \(K\) and small \(\gamma\) make the quasi-equilibrium region small. If we are
to approach the quasiequilibrium region to the point $x_c=0$ using the PID controller, irrespective of the constant parameter vector $\theta$ and desired configurations $(q_d,q_d,q_d)$, then one of three conditions should be satisfied: the one is that $K$ gain matrix goes to infinity, another is that $L_2$-gain $\gamma$ to zero and the other is that the gain $K$ to infinity. This explains indirectly the reason why the PID controller for trajectory tracking model of Lagrangian system can not bring the global asymptotic stability (GAS). In fact, the quasiequilibrium region of Theorem 2 has very close relation with performance limitation of the PID controller. Until now, the quasiequilibrium region can be used as a criterion for target performance chosen by user. Also, it indirectly proves the existence of gains which can achieve the target performance. In the following section, we will propose an automatic performance tuning method of the PID controller assisting to accomplish the target performance.

3.3 Autotuning Law. Since the quasiequilibrium region is determined by the size of gains of PID controller and the inverse dynamics $Y_s(t)\theta$, we should know the regressor matrix $Y_s(t)$ and plant parameter vector $\theta$ to calculate the quasiequilibrium region. Since it is difficult to exactly identify the parameters of Lagrangian systems, conversely, we will use the quasiequilibrium region as a criterion of target performance. For example, if the target performance is determined, then the size of the quasiequilibrium region should be adjusted so that it can achieve the target performance. Therefore, the autotuning law for gains of the PID controller, is required to adjust it. In this respect, we choose the gain matrix $K$ in (21) as an autotuning parameter. Finally, the autotuning law is derived from the direct adaptive control scheme, based on ISS characteristics of trajectory tracking system, in the following Theorem.

Theorem 3: Let $s=\dot{x}+K_p e+K_i \int e dt$. Assume that there exists the smallest constant diagonal gain matrix $K_{\Omega}>0$ accomplishing the target performance ($\Omega$) as follows:

$$\sup_{0\leq t\leq T} |s(t)| \leq \Omega.$$ 

For $K_{\Omega}>\dot{K}(t)$, if the autotuning PID controller:

$$\tau=(\dot{K}(t)+\gamma^2 I)s,$$

using the autotuning law as the following form:

$$\frac{d\dot{K}_i}{dt}=\Gamma_s \dot{x}_s^2(t), \quad \text{for} \quad i=1,\ldots,n$$

is applied to the trajectory tracking system (5), then the closed-loop control system is extended disturbance input-to-state stable (ISS), where $s_i$ is the $i$th element of composite error vector $s$, $\dot{K}_i$ and $\Gamma_i$ are $i$th diagonal elements of the diagonal time-varying matrix $\dot{K}(t)>0$ and the update gain matrix $\Gamma>0$, respectively.

Proof: First, we take Lyapunov function as the following form:

$$V(s, \dot{K}, \dot{\dot{K}})=\frac{1}{2}s^T M s+\frac{1}{2}\text{tr}[(\dot{K}(t)-K_0)\Gamma^{-1}(\dot{K}(t)-K_0)],$$

where $\text{tr}[]$ means the trace of given matrix. If the autotuning PID controller (22) is applied to the trajectory tracking system model (5), then we can get the time derivative of Lyapunov function along the solution trajectory of closed-loop system as follows:

$$\dot{V}=-s^T\left(K+\frac{1}{2}\gamma^2 I\right)s-\frac{\gamma^2}{2}\left|s-w\right|^2+\frac{\gamma^2}{2}|w|^2$$

$$+\text{tr}[(\dot{K}(t)-K_0)\Gamma^{-1}\dot{K}(t)]$$

$$=-s^T\left(K_\Omega+\frac{1}{2}\gamma^2 I\right)s-\frac{\gamma^2}{2}\left|s-w\right|^2+\frac{\gamma^2}{2}|w|^2-s^T(\dot{K}(t)-K_\Omega)s$$

$$+\text{tr}[(\dot{K}(t)-K_\Omega)\Gamma^{-1}\dot{K}(t)].$$

Here, if the following matrix trace property is applied to the above equation:

$$s^T(\dot{K}(t)-K_\Omega)s=\text{tr}[(\dot{K}(t)-K_\Omega)ss^T],$$

then above time derivative of Lyapunov function is rearranged as follows:

$$\dot{V}=-s^T\left(K_\Omega+\frac{1}{2}\gamma^2 I\right)s-\frac{\gamma^2}{2}\left|s-w\right|^2+\frac{\gamma^2}{2}|w|^2+\text{tr}[(\dot{K}(t)-K_\Omega)$$

$$\times(\Gamma^{-1}\dot{K}(t)-ss^T)].$$

(25)

Also, if the diagonal elements of $(\Gamma^{-1}\dot{K}(t)-ss^T)$ are zeros, then the trace term of (25) becomes zero because $(\dot{K}(t)-K_\Omega)$ is a diagonal matrix. In other words, the autotuning law (23) is derived from the following relation:

$$\frac{d\dot{K}_i}{dt}=\Gamma_s \dot{x}_s^2(t), \quad \text{for} \quad i=1,\ldots,n$$

then $\text{tr}[(\dot{K}(t)-K_\Omega)(\Gamma^{-1}\dot{K}(t)-ss^T)]=0$.

Therefore, if the autotuning PID controller (22) is applied to trajectory tracking system, then we can get the following relation from (25):

$$\dot{V}=-s^T\left(K_\Omega+\frac{1}{2}\gamma^2 I\right)s+\frac{\gamma^2}{2}|w|^2.$$ 

(26)

Also, since the right-hand sides of (26) are unbounded functions for $s$ and $w$, respectively, the above inequality follows the ISS characteristics of (2). Hence, the trajectory tracking system with an autotuning PID controller is extended disturbance input-to-state stable (ISS).

Actually, if the PID controller cannot achieve the target perfor-
Theorem 4: Assume that the update gain matrix $\Gamma$ and an initial $\hat{K}(0)$ are determined sufficiently large to satisfy the following inequality:

$$\hat{K}(t) > \frac{1}{2} Y(x,t) Y^T(x,t).$$

If the autotuning PID controller (22) is applied to the trajectory tracking system model (5), then its composite error is upper bounded as the following form:

$$|s(t)| \leq |s(0)| e^{-\frac{(1/\gamma^2)\gamma^2 t}{2}} + \gamma^2 |d| + \gamma |\Theta|.$$  

where $s(0)$ is an initial composite error, $\gamma$ is the maximum eigenvalue of $M$, and $\Theta$ is the real parameter vector of the Lagrangian system.

Proof: If the autotuning PID controller (22) is applied to the trajectory tracking system (5) using the extended disturbance (16) divided into linear parameterization part and external disturbance, then we can get the time derivative of positive real-valued function $\left(\frac{1}{2} s^T M s\right)$ along the solution trajectory of closed-loop control system as follows:

$$\frac{d}{dt} \left(\frac{1}{2} s^T M s\right) = -s^T \left(\hat{K} + \frac{1}{\gamma^2} I\right)s + s^T Y \Theta + s^T d$$

$$= -\frac{1}{2\gamma^2} |s|^2 + \frac{\gamma^2}{2} |s|^2 - d^2 + \frac{\gamma^2}{2} |d|^2$$

$$= -s^T \left(\hat{K} - \frac{1}{2} Y Y^T\right)s + \frac{\gamma^2}{2} |s|^2 - \frac{1}{2} |Y|^2 - \frac{1}{2} |\Theta|^2$$

$$\leq -\frac{1}{2\gamma^2} |s|^2 - s^T \left(\hat{K} - \frac{1}{2} Y Y^T\right)s + \frac{\gamma^2}{2} |s|^2 + \frac{1}{2} |\Theta|^2.$$  

Here, if we use the assumption (28) and the maximum eigenvalue of inertia matrix $M$, then above equation can be simplified to

$$\frac{d}{dt} \left(\frac{1}{2} s^T M s\right) \leq \frac{\gamma^2}{2} |d|^2 + \frac{1}{2} |\Theta|^2.$$  

If we multiply above inequality by $e^{(1/\gamma^2)\gamma^2 t}$, then it becomes

$$\frac{d}{dt} \left(\frac{1}{2} s(t)^2 e^{(1/\gamma^2)\gamma^2 t}\right) \leq \frac{\gamma^2}{2} |d(t)|^2 e^{(1/\gamma^2)\gamma^2 t} + \frac{1}{2} |\Theta|^2 e^{(1/\gamma^2)\gamma^2 t}.$$  

Integrating (30) over $[0,t]$, we arrive at the following form:

$$|s(t)|^2 \leq |s(0)|^2 e^{(1/\gamma^2)\gamma^2 t} + \frac{\gamma^2}{\lambda} \int_0^t e^{(1/\gamma^2)\gamma^2 (\tau - \tau')} |d(\tau)|^2 d\tau$$

$$+ \frac{1}{\lambda} \int_0^t e^{(1/\gamma^2)\gamma^2 (\tau - \tau')} |d(\tau)|^2 d\tau$$

$$\leq |s(0)|^2 e^{(1/\gamma^2)\gamma^2 t} + \frac{\gamma^2}{\lambda} \sup_{\tau \geq t} |d(\tau)|^2 + \frac{1}{\lambda} \int_0^t e^{(1/\gamma^2)\gamma^2 (\tau - \tau')} |d(\tau)|^2 d\tau.$$
Using the fact that $\sqrt{a^2+b^2+c^2} \leq |a| + |b| + |c|$, we obtain an explicit upper bound for composite error as follows:

$$|s(t)| \leq |s(0)|e^{-\lambda t} + \gamma_\theta |\theta|.$$  

In above Theorem, the initial composite error can be set to zero by making the desired trajectory smooth. Hence, the composite error is bounded by the size of plant parameter vector and $L_\infty$ norm of external disturbance as follows:

$$|s(t)| \leq \gamma_\theta |\theta|,$$

under the assumption (28). However, we do not know whether the assumption is satisfied or not. In the assumption, the regressor matrix $Y(x,t)$ is dependent on desired configurations and errors. The autotuned gain matrix $\hat{K}(t)$ is affected by the update gain matrix $\Gamma$ and an initial $\hat{K}(0)$. Therefore, an initial $\hat{K}(0)$ should satisfy at least the following inequality:

$$\hat{K}(0) > \frac{1}{\gamma}Y(0,0)Y^T(0,0).$$

As a matter of fact, since $Y(0,0)$ is equal to the simple regressor obtained by separating parameter vector from gravity torque $g(q)$, if there exists no gravitational torque, namely, $g(q) = 0$, then $\hat{K}(0) > 0$. Also, the update gain matrix should be chosen sufficiently large to satisfy the assumption because it determines the increasing rate of autotuned gains.

If there are no external disturbances $d = 0$, then upper bound of composite error is bounded only by the size of parameter vector of Lagrange system:

$$|s(t)| \leq \gamma_\theta |\theta|.$$  

As a matter of fact, above boundedness of composite error means the performance limitation of an autotuning PID controller. Also, we can see in (31) that the composite error can be adjusted by $L_\infty$-gain $\gamma$ like a linear tuning rule explained in Remark 2. Basically, the autotuning PID controller has the performance limitation for trajectory tracking model of the Lagrangian system. Hence, the model adaption is required to overcome this limitation and bring the global asymptotic stability (GAS).

3.6 Model Adaptation. First of all, the regressor should be obtained to use the model adaption in controller. However, it is not easy to obtain the applicable regressor for general Lagrangian systems because the plant parameter vector can be maximum $10n$-dimensional one $\theta \in \mathbb{R}^{10n}$, according to the report in [21]. This is a serious disadvantage for the controller using a model adaptation. However, if the regressor can be obtained anyhow, then the adaptive motion control scheme can be applied to the trajectory tracking system. Also, the controller using model adaptation overcomes the performance limitation of the PID control itself as suggested in the following Theorem.

**Theorem 5:** Assume that there exists an autotuning PID controller (22) in Theorem 3. If the adaptive autotuning PID controller as the following form:

$$\tau = \left(\hat{K}(t) + \frac{1}{\gamma}I\right)s + Y(x,t)\hat{\theta}(t)$$

with the plant parameter update law:

$$\hat{\theta}(t) = \Lambda^{-1}Y^T(x,t)s$$

is applied to the trajectory tracking system (5), then the closed-loop control system is external disturbance input-to-state stable (ISS), where $\Lambda$ is the gain matrix for parameter update.

**Proof:** First, let us consider a Lyapunov function as follows:

$$V_{\text{Adaptive}}(s,\hat{K},\hat{\theta},t) := V(s,\hat{K},t) + \frac{1}{2}(\theta - \hat{\theta}(t))^T \Lambda (\theta - \hat{\theta}(t)).$$

where $\Lambda$ is a constant, symmetric and positive definite matrix and $V(s,\hat{K},t)$ is (24). If the adaptive autotuning PID controller (32) is applied to the trajectory tracking system (5), then we get the time derivative of Lyapunov function (34) along the solution trajectory of closed-loop system with the control law (32) as follows:

$$\dot{V}_{\text{Adaptive}} = -s^T(\hat{K}(t) + \frac{1}{\gamma}I)s + s^Td + s^TY(\theta - \hat{\theta}(t))$$

$$+ \text{tr}[(\hat{K}(t) - K_{\Omega})^{-1} \hat{K}(t)] - (\theta - \hat{\theta}(t))^T \Lambda \hat{\theta},$$

$$= -s^T\left[K_{\Omega} + \frac{1}{2\gamma}I\right]s - \frac{\gamma^2}{2} s - d + \frac{\gamma^2}{2} |d|^2$$

$$+ \text{tr}[(\hat{K}(t) - K_{\Omega})^{-1} \hat{K}(t) - ss^T]$$

$$+ (\theta - \hat{\theta}(t))^T(Y^TS - \Lambda \hat{\theta}).$$

If the autotuning law (23) and parameter update law (33) are applied to above (35), then we can get the following similar to (2):

$$\dot{V}_{\text{Adaptive}} \leq -s^T\left[K_{\Omega} + \frac{1}{2\gamma}I\right]s - \frac{\gamma^2}{2} |d|^2.$$  

Since the right-hand side of above inequality (36) is unbounded function for $s$ and $d$, respectively, hence, we conclude that the adaptive autotuning PID controller brings the external disturbance input-to-state stability (ISS).

Additionally, if there exists no external disturbances, in other words, $d = 0$, then we can see that the adaptive auto-tuning PID controller offers the global asymptotic stability (GAS) for the trajectory tracking model of Lagrangian system. However, this adaptive plus autotuning PID control scheme can be applied only when the regressor is known. For most Lagrangian systems, the PID control scheme has been used without using model adaptation because it was difficult or impossible to obtain the exact regressor for many Lagrangian systems. Therefore, only an autotuning PID controller (22) will be implemented to show the validity of automatic performance tuning method in following section.

4 Experimental Results

To show the effectiveness of an autotuning PID controller, we employ three link robotic manipulator as the representative Lagrangian system. This robotic manipulator as shown in Fig. 6...
consists of three direct drive motors: the first axis motor has 200 [N m] capability, the second one 55 [N m] and the third one 18 [N m]. The desired configuration profiles of Fig. 7(a) are obtained by solving inverse kinematics for 3 line segments whose lengths are all 0.7 [m]. Also, the given trajectories require the fast motion (maximum velocity =3 [rad/s]) of robotic manipulator as shown in Fig. 7(b). First, we determine the static gains of autotuning PID controller (22) as \(K_p=20\), \(K_i=100\), and \(\gamma=0.5\) satisfying the design guidelines (C1),(C2),(C3). Now, the controller has the following form: for \(i=1,2,3\),

\[
\tau_i = (\hat{K}_i(t) + 4)s_i
\]

\[
s_i = 20\dot{e}_i + 100 \int e_i dt
\]

\[
\frac{d\hat{K}_i(t)}{dt} = \Gamma s_i^2, \quad \text{if} \quad |s_i| > \frac{\Omega}{\sqrt{2n}}
\]

where \(\tau_i\) is the \(i\)th element of input torque vector \(\tau\) and \(n=3\). Also, initial auto-tuned gains are determined as \(\hat{K}_i(0)=0.1\) because the robotic manipulator is not affected by the gravity. Second, since the composite error is approximately proportional to the configuration error with proportional constant \(K_p\), the target performance can be approximately determined as follows:

\[
\Omega = \sqrt{2n} \times |s_i| = \sqrt{2n} \times K_p \times |e_i|.
\]

where \(|s_i|\) and \(|e_i|\) are the target composite error and configuration error, respectively. For instance, if we are to obtain the performance of \(|e_i|<0.02\) [rad] for each driving axis, then the target performance should be determined as \(\Omega=1.0\) by (37). Also, the update gain \(\Gamma=1000\) is used in an autotuning law. Figures 7(c) and 7(d) show experimental results: the configuration error and its velocity error. In figures, the errors are large at initial time, however, they are reduced till the target performance is achieved by an autotuning law.

As a matter of fact, the autotuning law is executed at the exterior of two dotted lines in Fig. 8(a). After the autotuning process is finished, the autotuned gains arrive at \(\hat{K}_1=136.98, \hat{K}_2=65.60,\) and \(\hat{K}_3=6.83\) as we can see in Fig. 8(c). To examine the autotuning process in detail, the horizontal ranges of 0 – 1 s in Figs. 8(a) and 8(c) are enlarged as shown in Figs. 8(b) and 8(d). The autotuning for the first axis is started at 0.11 s and ended at 0.26 s because the error goes over the criterion (dotted line: \(1/\sqrt{2}=0.408\)) for the first time as shown in Fig. 8(b). Also, the error of second axis goes over the criterion between 0.13 and 0.24 s. Finally, since the error of third axis goes over the criterion downward twice, the autotuning process is implemented twice as shown in Fig. 8(d). The experimental result of Fig. 8(a) shows that the target performance is achieved after 0.6 s when the autotuning is finished.

5 Concluding Remarks

First, the performance limitation of the PID controller was analyzed for the trajectory tracking model of the Lagrangian system. The passive performance tuning method was obtained using the performance limitation. Second, the quasiequilibrium region was newly defined to guarantee the existence of the PID controller achieving target performance. Finally, we proposed the autotuning PID controller achieving target performance. The autotuning law was derived from the direct adaptive control scheme based on the ISS and the analysis result of performance limitation. Through the experiment, we showed the effectiveness of the automatic performance tuning method.
Acknowledgments

This research was supported in part by a grant (02-PJ3-P6- EV04-0003) of Ministry of Health and Welfare, by the International Cooperation Research Program (M6-0302-00-0009-03- A01-00-004-00) of the Ministry of Science and Technology, by the National Research Laboratory (NRL) Program (M1-0302-00- 0040-03-J00-00-024-00) of the Ministry of Science and Technology, and by a grant (M1-0214-00-0116) of the Ministry of Science and Technology Republic of Korea.

References


