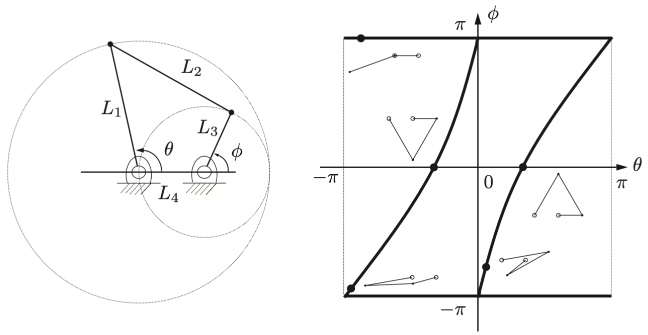

Figure 7.5: (Left) A planar four-bar linkage and (right) its one-dimensional C-space, represented in bold in the θ–ϕ space. Also shown on the right are five sample configurations (bold dots), three of which are near bifurcation points and two of which are far removed from a bifurcation point.

3 Singularities

- Let us highlight the essential features of closed chain singularities via two planar examples: a four-bar linkage and a five-bar linkage.
- Closed-chain singularities are divided into three basic types: actuator singularities, configuration space singularities, and end-effector singularities.
- Projecting the C-space onto the joint angles (θ, ϕ) leads to the bold curve in the Figure.
- In terms of θ and ϕ, the kinematic loop constraint equations for the four-bar linkage can be expressed as

$$\phi = \tan^{-1}\frac{\beta}{\alpha} \pm \cos^{-1}\frac{\gamma}{\sqrt{\alpha^2+\beta^2}} \qquad \text{Exercise 2.25}$$

- The existence and uniqueness of solutions to the equations above depend on the link lengths $L_1, \cdots, L_4$. A distinctive feature of Figure is the presence of bifurcation points where branches of the curve meet.

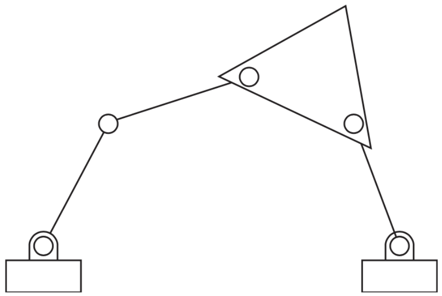

Figure 7.6: A planar five-bar linkage.

- Consider five-bar linkage. The kinematic loop constraint equations can be written

$$L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) + \cdots + L_4 \cos(\theta_1+\theta_2+\theta_3+\theta_4) = L_5$$
$$L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) + \cdots + L_4 \sin(\theta_1+\theta_2+\theta_3+\theta_4) = 0$$

- Writing these two equations in the form $f(\theta_1,\cdots,\theta_4)=0$, where $f:\Re^4\to\Re^2$, the configuration space can be regarded as a two-dimensional surface in $\Re^4$.
- Like the bifurcation points of the four-bar linkage, self-intersections of the surface can also occur.
- At such points the constraint Jacobian loses rank. For the five-bar linkage, any point θ at which

$$\text{rank}\ \frac{\partial f}{\partial \theta}(\theta) < 2$$

corresponds to what we call a configuration space singularity.

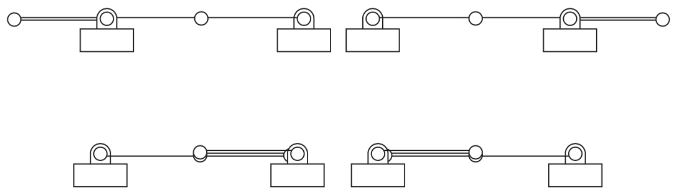

Figure 7.7: Configuration space singularities of the planar five-bar linkage.

- Figure illustrates the possible configuration space singularities of the five-bar linkage.
- Notice that so far we have made no mention of which joints of the five-bar linkage are actuated, or where the end-effector frame is placed.
- The notion of a configuration space singularity is completely independent of the choice of actuated joints or where the end-effector frame is placed.

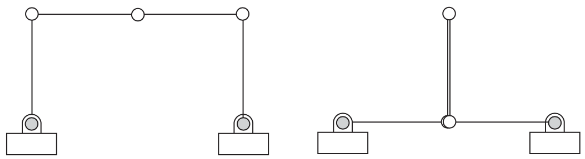

Figure 7.8: Actuator singularities of the planar five-bar linkage, where in each case the two actuated joints are shaded gray. The singularity on the left is nondegenerate, while the singularity on the right is degenerate.

- We now consider the case when two joints of the five-bar linkage are actuated. Referring to Figure, the two revolute joints fixed to ground are the actuated joints.
- Under normal operating conditions, the motions of the actuated joints can be independently controlled. Alternatively, locking the actuated joints should immobilize the five-bar linkage and turn it into a rigid structure.
- For the nondegenerate actuator singularity shown in the left-hand panel of Figure, rotating the two actuated joints oppositely and outward will pull the mechanism apart; rotating them oppositely and inward would either crush the inner two links or cause the center joint to unpredictably buckle upward or downward.
- For the degenerate actuator singularity shown on the right, even when the actuated joints are locked in place the inner two links are free to rotate.
- The reason for classifying these singularities as actuator singularities is that, by relocating the actuators to a different set of joints, such singularities can be eliminated.
- For both the degenerate and nondegenerate actuator singularities of the five-bar linkage, relocating one actuator to one of the other three passive joints eliminates the singularity.

- Actuator singularities can be characterized mathematically by the rank of the constraint Jacobian. As before, write the kinematic loop constraints in differential form:

$$H(q)\dot{q} = \begin{bmatrix} H_a(q) & H_p(q) \end{bmatrix} \begin{bmatrix} \dot{q}_a \\ \dot{q}_p \end{bmatrix} = 0$$

- With the above definitions, we have the following:
  - If rank $H_p(q) < p$ then q is an actuator singularity.
  - If rank $H(q) < p$ then q is a configuration space singularity. Note that under this condition $H_p(q)$ is also singular (the converse is not true, however). The configuration space singularities can therefore be regarded as the intersection of all possible actuator singularities obtained over all possible combinations of actuated joints.

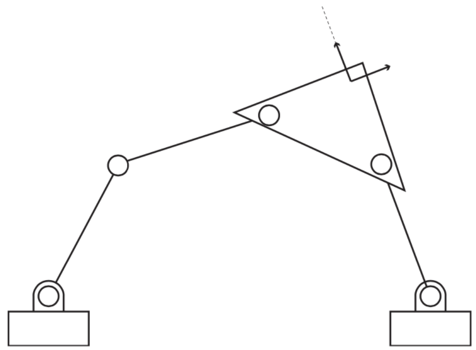

Figure 7.9: End-effector singularity of the planar five-bar linkage.

- The final class of singularities depends on the choice of end-effector frame.
- Figure shows the five-bar linkage in an end-effector singularity for a given choice of end-effector location.
- Since the two links of the 2R robot are aligned, the end-effector frame can have no component of motion along the direction of the links.
- End-effector singularities are independent of the choice of actuated joints.
- Choose any valid set of actuated joints q_a such that the mechanism is not at an actuator singularity.

4 Homework : Chapter 7

- Please solve and submit Exercise 7.1, 7.3, 7.4, 7.7, 7.8 till May 28 (upload it as a pdf form or email me)