5 Force Control

- Pure force control is only possible if the environment provides resistance forces in every direction (e.g., if the end-effector is embedded in concrete or attached to a spring providing resistance in every motion direction).

- Let $\mathcal{F}_{tip}$ be the wrench applied by the manipulator to the environment. The manipulator dynamics can be written as

$$M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + b(\dot{\theta}) + J^T(\theta)\mathcal{F}_{tip} = \tau$$

  where $\mathcal{F}_{tip}$ and $J(\theta)$ are defined in the same frame (the space frame or the endeffector frame).

- Assume that the robot typically moves slowly (or not at all) during a force control task, then we can ignore the acceleration and velocity terms to get

$$g(\theta) + J^T(\theta)\mathcal{F}_{tip} = \tau$$

  1. In the absence of any direct measurements of the force-torque at the robot end-effector, joint-angle feedback alone can be used to implement the force control law (desired force feedforward control)

$$\tau = \widetilde{g}(\theta) + J^T(\theta)\mathcal{F}_d$$

     where $\mathcal{F}_d$ is the desired wrench. In the case of a DC electric motor without gearing, torque control can be achieved by current control of the motor.

  2. Another solution is to equip the robot arm with a six-axis force-torque sensor between the arm and the end-effector to directly measure the end-effector wrench $\mathcal{F}_{tip}$. Add a PI force

controller to a feedforward term

$$\tau = \widetilde{g}(\theta) + J^T(\theta)\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t}\right)$$

where $\mathcal{F}_e = \mathcal{F}_d - \mathcal{F}_{tip}$ and K_{fp} and K_{fi} are positive-definite proportional and integral gain matrices, respectively.

- In the case of a nonzero but constant force disturbance, arising from an incorrect model of $\widetilde{g}(\theta)$, for example, we take the derivative to get

$$K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t} = c \qquad \rightarrow \qquad K_{fp}\dot{\mathcal{F}}_e + K_{fi}\mathcal{F}_e = 0$$

  showing that $\mathcal{F}_e$ converges to zero for positive-definite K_{fp} and K_{fi}.

3. Since a typical force-control task requires little motion, we can limit this acceleration by adding velocity damping. This gives the modified control law

$$\tau = \widetilde{g}(\theta) + J^T(\theta)\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t} - K_{damp}\mathcal{V}\right)$$

where K_{damp} is positive definite.

6 Hybrid Motion-Force Control

- Most tasks requiring the application of controlled forces also require the generation of controlled motions.
- If the the task space is n-dimensional then we are free to specify n of the $2n$ forces and motions at any time t; the other n are determined by the environment.
- Apart from this constraint, we also should not specify forces and motions in the "same direction," as then they are not independent.

6.1 Natural and Artificial Constraints

- A particularly interesting case occurs when the environment is infinitely stiff (rigid constraints) in k directions and unconstrained in $n - k$ directions.

- In this case, we cannot choose which of the $2n$ motions and forces to specify - the contact with the environment chooses the k directions in which the robot can freely apply forces and the $n-k$ directions of free motion.

- For example, consider a task space with the $n = 6$ dimensions of $SE(3)$.
  - Then a robot firmly grasping a cabinet door has $6 - k = 1$ motion freedom of its end-effector, i.e., rotation about the cabinet hinges,
  - $k = 5$ force freedoms; the robot can apply any wrench that has zero moment about the axis of the hinges without moving the door.

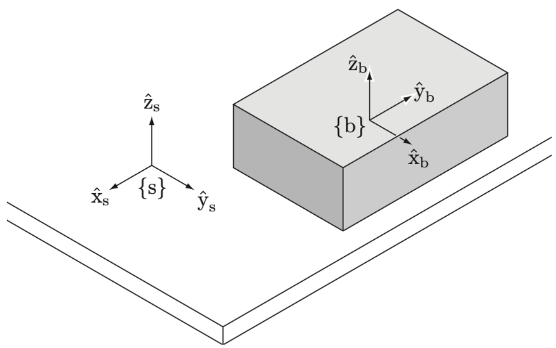

Figure 11.22: The fixed space frame {s} is attached to the chalkboard and the body frame {b} is attached to the center of the eraser.

- As a final example, consider a robot erasing a frictionless chalkboard using an eraser modeled as a rigid block
- Let $X_{sb}(t) \in SE(3)$ be the configuration of the block's frame {b} relative to a space frame {s}.
- The body-frame twist and wrench are written $\mathcal{V}_b = (\omega_x, \omega_y, \omega_z, v_x, v_y, v_z)$ and $\mathcal{F}_b = (m_x, m_y, m_z, f_x, f_y, f_z)$, respectively.
- Maintaining contact with the board puts $k = 3$ constraints on the twist: $\omega_x = 0, \omega_y = 0, v_z = 0$. These constraints are called natural constraints, specified by the environment. There are $6-k = 3$ natural constraints on the wrench, too: $m_z = f_x = f_y = 0$.
- In light of the natural constraints, we can freely specify any twist of the eraser satisfying the $k = 3$ velocity constraints and any wrench satisfying the $6 - k = 3$ wrench constraints (provided that $f_z < 0$, to maintain contact with the board). These motion and force specifications are called artificial constraints.
- The artificial constraints cause the eraser to move with $v_x = k_1$ while applying a constant force k_2 against the board. (refer to the table below figure 11.22)

6.2 A Hybrid Motion-Force Controller

- If the environment is rigid, then we can express the k natural constraints on the velocity in task space as the Pfaffian constraints

  $$A(\theta)\mathcal{V} = 0$$

  where $A(\theta) \in \Re^{k\times 6}$ for twists $\mathcal{V} \in \Re^6$.

- If the task-space dynamics of the robot (Section 8.6), in the absence of constraints, is given by

  $$\mathcal{F} = \Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V})$$

  where $\tau = J^T(\theta)\mathcal{F}$ are the joint torques and forces created by the actuators, then the constrained dynamics is

  $$\mathcal{F} = \Lambda(\theta)\dot{\mathcal{V}} + \eta(\theta, \mathcal{V}) + A^T(\theta)\lambda \qquad \rightarrow \qquad \dot{\mathcal{V}} = \Lambda^{-1}[\mathcal{F} - \eta - A^T\lambda]$$

  where $\lambda \in \Re^k$ are Lagrange multipliers.

- The requested wrench $\mathcal{F}_d$ must lie in the column space of $A^T(\theta)$.

- Since the constraint must be satisfied at all times, we can replace it by the time derivative

  $$A(\theta)\dot{\mathcal{V}} + \dot{A}(\theta)\mathcal{V} = 0 \qquad \rightarrow \qquad A(\theta)\dot{\mathcal{V}} = -\dot{A}(\theta)\mathcal{V}$$

- The constraint is rearranged into

  $$A\Lambda^{-1}[\mathcal{F} - \eta - A^T\lambda] = -\dot{A}\mathcal{V} \qquad \rightarrow \qquad A\Lambda^{-1}A^T\lambda = A\Lambda^{-1}[\mathcal{F} - \eta] + \dot{A}\mathcal{V}$$

and the Lagrange multiplier is

$$\lambda = (A\Lambda^{-1}A^T)^{-1}[A\Lambda^{-1}(\mathcal{F} - \eta) - A\dot{\mathcal{V}}]$$

- If λ is applied to the constrained dynamics, then we have $n - k$ independent motion equations

$$\begin{aligned} \mathcal{F} &= \Lambda\dot{\mathcal{V}} + \eta + A^T(A\Lambda^{-1}A^T)^{-1}[A\Lambda^{-1}(\mathcal{F} - \eta) - A\dot{\mathcal{V}}] \\ [I - A^T(A\Lambda^{-1}A^T)^{-1}A\Lambda^{-1}]\mathcal{F} &= [I - A^T(A\Lambda^{-1}A^T)^{-1}A\Lambda^{-1}](\Lambda\dot{\mathcal{V}} + \eta) \\ P(\theta)\mathcal{F} &= P(\theta)(\Lambda\dot{\mathcal{V}} + \eta) \end{aligned}$$

where

$$P(\theta) = I - A^T(A\Lambda^{-1}A^T)^{-1}A\Lambda^{-1}$$

  - The $n \times n$ matrix $P(\theta)$ has rank $n - k$ and projects an arbitrary manipulator wrench $\mathcal{F}$ onto the subspace of wrenches that move the end-effector tangent to the constraints.
  - The rank-k matrix $I - P(\theta)$ projects an arbitrary wrench $\mathcal{F}$ onto the subspace of wrenches that act against the constraints.
  - Thus $P(\theta)$ partitions the n-dimensional force space into wrenches that address the motion control task and wrenches that address the force control task.
  - Since $PA^T = 0$ and $A\Lambda^{-1}P = 0$, P is null space projection matrix.
  - Since $(I - P)A^T = A^T$ is range space projection matrix.
  - If $\Lambda = I$, $P = I - A^T(AA^T)^{-1}A = I - A^+A$ and $I - P = A^T(AA^T)^{-1}A = A^+A$

- Our hybrid motion-force controller is simply the sum of a task-space motion controller, derived from the computed torque control law, and a task-space force controller, each projected to generate forces in its appropriate subspace.

$$\tau = J_b^T(\theta)\left(P(\theta)\widetilde{\Lambda}(\theta)\left(\frac{d}{dt}([Ad_{X_{sb}^{-1}X_{sd}}]\mathcal{V}_d) + K_pX_e + K_i\int X_e(\mathrm{t})dt + K_d\mathcal{V}_e\right)\right.$$
$$\left. + (I - P(\theta))\left(\mathcal{F}_d + K_{fp}\mathcal{F}_e + K_{fi}\int \mathcal{F}_e(\mathrm{t})d\mathrm{t}\right) + \widetilde{\eta}(\theta, \mathcal{V}_b)\right)$$

  - Because the dynamics of the two controllers are decoupled by the orthogonal projections P and $I - P$, the controller inherits the error dynamics and stability analyses of the individual force and motion controllers on their respective subspaces.
  - A difficulty in implementing the hybrid control law in rigid environments is knowing the form of the constraints $A(\theta)\mathcal{V} = 0$ active at any time. This is necessary to specify the desired motion and force and to calculate the projections, but any model of the environment will have some uncertainty.

7 Impedance Control

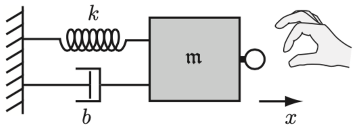

Figure 11.23: A robot creating a one-dof mass–spring–damper virtual environment. A human hand applies a force f to the haptic interface.

- Ideal motion control corresponds to high impedance (little change in motion due to force disturbances) while ideal force control corresponds to low impedance (little change in force due to motion disturbances).
- For example, a robot used as a haptic surgical simulator could be tasked with mimicking the mass, stiffness, and damping properties of a virtual surgical instrument in contact with virtual tissue.
- The dynamics for a one-dof robot rendering an impedance can be written

$$\mathrm{m}\ddot{x} + b\dot{x} + kx = f$$

where x is the position, m is the mass, b is the damping, k is the stiffness, and f is the force applied by the user.

- Loosely, we say that the robot renders high impedance if one or more of the m, b, k parameters, usually including b or k, is large. Similarly, we say that the impedance is low if all these parameters are small.
- Taking the Laplace transform, we get

$$(\mathrm{m}s^2 + bs + k)X(s) = F(s) \qquad \rightarrow \qquad Z(s) = \frac{F(s)}{X(s)} = ms^2 + bs + k \quad \textbf{impedance}$$

and the impedance is defined by the transfer function from position perturbations to forces, $Z(s) = \frac{F(s)}{X(s)}$.

- Thus impedance is frequency dependent, with a low-frequency response dominated by the spring and a high-frequency response dominated by the mass.
- The admittance, $Y(s)$, is the inverse of the impedance:

$$Y(s) = Z^{-1} = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

- The goal of impedance control is to implement the task-space behavior

$$M\ddot{x} + B\dot{x} + Kx = f_{ext}$$

where $x \in \Re^n$ is the task-space configuration in a minimum set of coordinates. The values of M, B, and K may change, depending on the location in the virtual environment, to represent objects

- The behavior could be implemented in terms of twists and wrenches instead, replacing f_{ext} by the (body or spatial) wrench $\mathcal{F}_{ext}$, $\dot{x}$ by the twist $\mathcal{V}$, $\ddot{x}$ by $\dot{\mathcal{V}}$, and x by the exponential coordinates $\mathcal{S}\theta$.
- There are two common ways to achieve the behavior.
  - The robot senses the endpoint motion $x(t)$ and commands joint torques and forces to create -f_{ext}, the force to display to the user. Such a robot is called impedance controlled, as it implements a transfer function $Z(s)$ from motions to forces. Theoretically, an impedance-controlled robot should only be coupled to an admittance-type environment.
  - The robot senses f_{ext} using a wrist force-torque sensor and controls its motions in response. Such a robot is called admittance controlled, as it implements a transfer function $Y(s)$ from forces to motions. Theoretically, an admittance-controlled robot should only be coupled to an impedancetype environment.

7.1 Impedance-Control Algorithm

- In an impedance-control algorithm, encoders, tachometers, and possibly accelerometers are used to estimate the joint and endpoint positions, velocities, and possibly accelerations.
- The impedance-controlled robots are not equipped with a wrist force-torque sensor and instead rely on their ability to precisely control joint torques to render the appropriate end-effector force -f_{ext}.
- A good control law might be

$$\tau = J^T(\theta)\left(\widetilde{\Lambda}(\theta)\ddot{x} + \widetilde{\eta}(\theta, \dot{x}) - (M\ddot{x} + B\dot{x} + Kx)\right)$$

- In the control law, it is assumed that $\ddot{x}, \dot{x}, x$ are measured directly. Measurement of the acceleration $\ddot{x}$ is likely to be noisy, and we eliminate $\widetilde{\Lambda}(\theta) = 0$ and set $M = 0$.
- The mass of the arm will be apparent to the user, but impedance-controlled manipulators are often designed to be lightweight.
- It is also not uncommon to assume small velocities and replace the nonlinear dynamics compensation with a simpler gravity-compensation model.
- Problems can arise when impedance controller is used to simulate stiff environments (the case of large K), because small changes in position, measured by encoders for example, lead to large changes in motor torques.

7.2 Admittance-Control Algorithm

- In an admittance-control algorithm the force f_{ext} applied by the user is sensed by the wrist load cell.
- A simple approach is to calculate the desired endeffector acceleration $\ddot{x}_d$ according to

$$M\ddot{x}_d + B\dot{x} + Kx = f_{ext}$$

  where $(x, \dot{x})$ is the current state.

- Solving, we get

$$\ddot{x}_d = M^{-1}(f_{ext} - B\dot{x} - Kx)$$

- For the Jacobian $J(\theta)$ defined by $\dot{x} = J(\theta)\dot{\theta}$, the desired joint accelerations $\ddot{\theta}_d$ can be solved as

$$\ddot{\theta}_d = J^{+}(\theta)(\ddot{x}_d - \dot{J}(\theta)\dot{\theta})$$

  and inverse dynamics used to calculate the commanded joint forces and torques τ.

- Simplified versions of this control law can be obtained when the goal is to simulate only a spring or a damper.
- To make the response smoother in the face of noisy force measurements, the force readings can be low-pass filtered.

8 Low-Level Joint Force/Torque Control

- Current Control of a Direct-Drive Motor
  - In this configuration, each joint has a motor amplifier and an electric motor with no gearhead.
  - The torque of the motor approximately obeys the relationship $\tau = k_t I$, i.e., the torque is proportional to the current through the motor.
  - The amplifier takes the requested torque, divides by the torque constant k_t, and generates the motor current I.
  - To create the desired current, a current sensor integrated with the amplifier continuously measures the actual current through the motor, and the amplifier uses a local feedback control loop to adjust the time-averaged voltage across the motor to achieve the desired current.
  - This local feedback loop runs at a higher rate than the control loop that generates the requested torques.
  - A typical example is 10 kHz for the local current control loop and 1 kHz for the outer control loop requesting joint torques.
- Current Control of a Geared Motor
  - This configuration is similar to the previous one, except that the motor has a gearhead.
  - A gear ratio $G > 1$ increases the torque available to the joint.
  - Advantage:
    * A smaller motor can provide the necessary torques.
    * The motor also operates at higher speeds, where it is more efficient at converting electrical power to mechanical power.
  - Disadvantage:
    * The gearhead introduces backlash (the output of the gearhead can move without the input moving, making motion control near zero velocity challenging) and friction.

    * Backlash can be nearly eliminated by using particular types of gearing, such as harmonic drive gears.
    * Friction, however, cannot be eliminated. The nominal torque at the gearhead output is Gk_tI, but friction in the gearhead reduces the torque available and creates significant uncertainty in the torque actually produced.

- Current Control of a Geared Motor with Local Strain Gauge Feedback
  - This configuration is similar to the previous one, except that the harmonic drive gearing is instrumented with strain gauges that sense how much torque is actually being delivered at the output of the gearhead.
  - This torque information is used by the amplifier in a local feedback controller to adjust the current in the motor so as to achieve the requested torque.
  - Advantage - Putting the sensor at the output of the gearing allows compensation of frictional uncertainties.
  - Disadvantage - There is additional complexity of the joint configuration.

- Series Elastic Actuator
  - A series elastic actuator (SEA) consists of an electric motor with a gearhead (often a harmonic drive gearhead) and a torsional spring attaching the output of the gearhead to the output of the actuator.
  - It is similar to the previous configuration, except that the torsional spring constant of the added spring is much lower than the spring constant of the harmonic drive gearing.
  - The angular deflectionof the spring is often measured by optical, magnetic, or capacitive encoders.
  - The spring's deflection is fed to a local feedback controller that controls the current to the motor so as to achieve the desired spring deflection, and therefore the desired torque.
  - Advantage

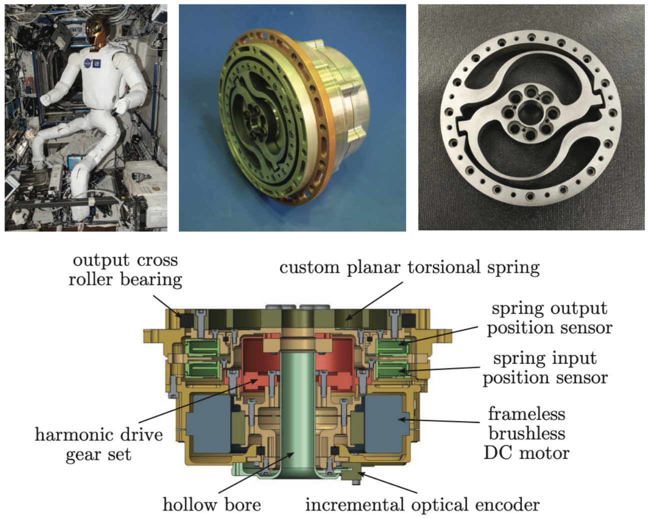

Figure 11.24: (Top left) The Robonaut 2 on the International Space Station. (Top middle) R2's hip joint SEA. (Top right) The custom torsional spring. The outer ring of hole mounts connects to the harmonic gearhead output, and the inner ring of hole mounts is the output of the SEA, connecting to the next link. The spring is designed with hard stops after approximately 0.07 rad of deflection. (Bottom) A cross-section of the SEA. The deflection $\Delta\phi$ of the torsional spring is determined by differencing the deflection readings at the spring input and the spring output. The optical encoder and spring deflection sensors provide an estimate of the joint angle. The motor controller–amplifier is located at the SEA, and it communicates with the centralized controller using a serial communication protocol. The hollow bore allows cables to pass through the interior of the SEA. All images courtesy of NASA.

    * The addition of the torsional spring makes the joint naturally soft, and therefore well suited for human-robot interaction tasks.
    * It also protects the gearing and motor from shocks at the output, such as when the output link hits something hard in the environment.
- Disadvantage
    * There is additional complexity of the joint configuration.
    * Also, the added dynamics due to the softer spring make it more challenging to control high-speed or high-frequency motions at the output.
- In 2011, NASA's Robonaut 2 (R2) became the first humanoid robot in space, performing operations on the International Space Station. Robonaut 2 incorporates a number of SEAs.

9 Homework : Chapter 11 (Not mandatory)

- Please solve and submit Exercise 11.1, 11.2, 11.4, 11.5, 11.6, 11.9, 11.17, if possible. (upload it as a pdf form or email me)