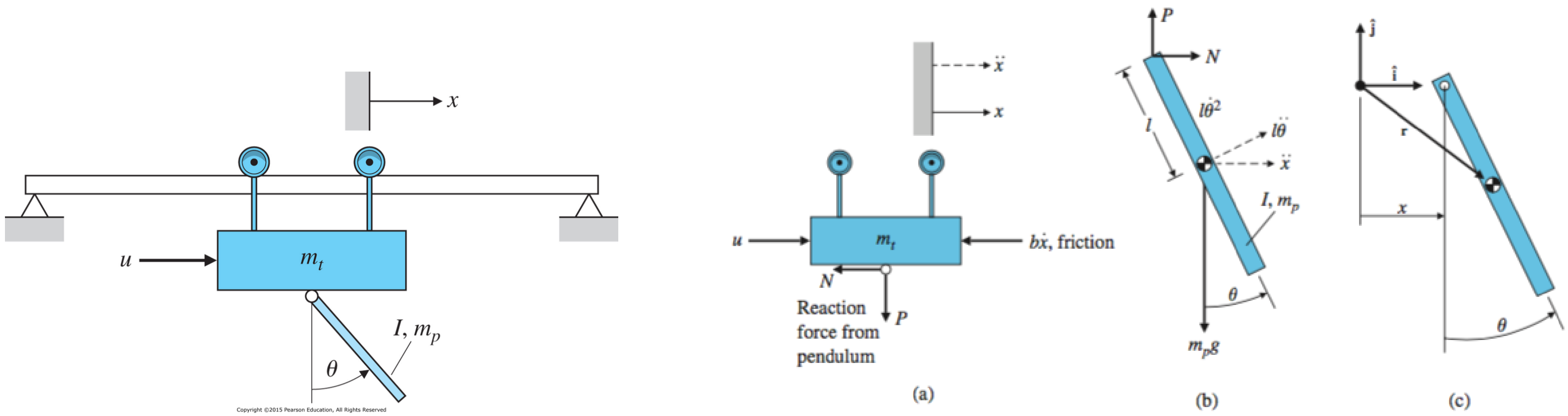
3. Combined Rotation and Translation

- The procedures to obtain the equation of motion are listed:
  (1) sketch the free-body diagrams
  (2) define coordinates and positive directions
  (3) determine all forces and moments acting
  (4) apply Newton's law and Euler's law
  (5) combine the equations to eliminate internal forces
  (6) check whether or not the number of indepdendent equations is equal to the number of unknowns.



- (Example 2.8, Hanging Crane) Assume that the moment of inertia I is denoted in terms of the mass center. For the pendulum,

$$N = m_p\ddot{x} + m_p l\ddot{\theta}\cos\theta - m_p l\dot{\theta}^2\sin\theta$$

$$P - m_p g = m_p l\ddot{\theta}\sin\theta + m_p l\dot{\theta}^2\cos\theta$$

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \qquad \rightarrow \qquad (I + m_p l^2)\ddot{\theta} + m_p gl\sin\theta = -m_p\ddot{x}l\cos\theta$$

For the crane,

$$m_t\ddot{x} = u - N - b\dot{x} \qquad \rightarrow \qquad (m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta}\cos\theta - m_p l\dot{\theta}^2\sin\theta = u$$

As a result, we have the complete equation of motion:

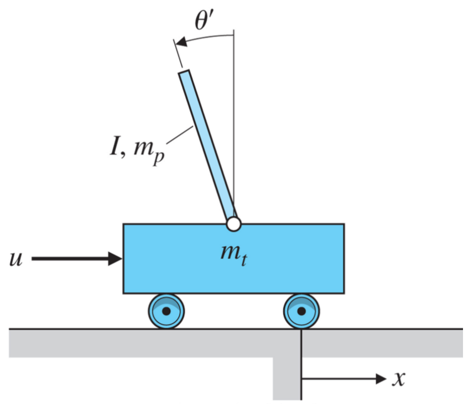
$$(I + m_p l^2)\ddot{\theta} + m_p gl\sin\theta + m_p\ddot{x}l\cos\theta = 0$$
$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta}\cos\theta - m_p l\dot{\theta}^2\sin\theta = u$$

For the linearization with small angle variation, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ and $\dot{\theta}^2 \approx 0$, we have

$$(I + m_p l^2)\ddot{\theta} + m_p gl\theta + m_p l\ddot{x} = 0$$
$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta} = u$$

Furthermore, ignoring the damping b, we can get the TF as follow:

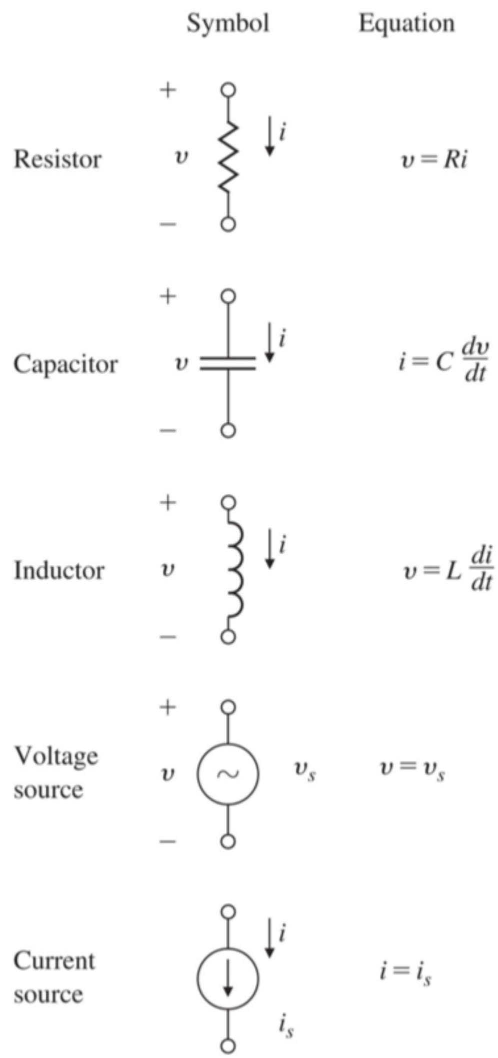
$$\frac{\Theta(s)}{U(s)} = \frac{-m_p l}{[(m_t + m_p)(I + m_p l^2) - m_p^2 l^2]s^2 + (m_t + m_p)m_p gl}$$



- (Homework #1, Due : March 29th, 2020) Derive the equation of motion (for the inverted pendulum on a cart (Segway)) aa the complete version and the linearized version.?

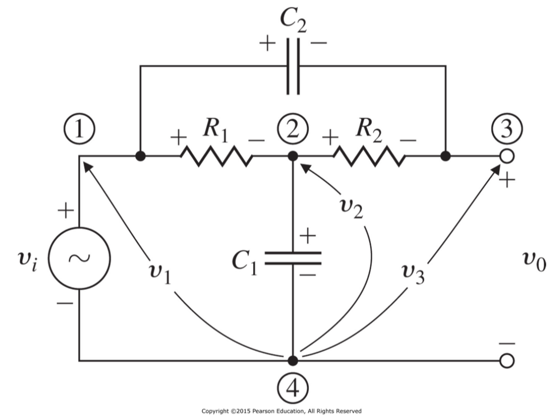
2 Models of Electrical Circuits

1. Consider the elements of electric circuits such as resistor, capacitor, inductor, voltage source, and current source.



2. (Kirchhoff's current law, KCL) The algebraic sum of currents leaving a junction or node equals the algebraic sum of currents entering that node.

3. (Kirchhoff's voltage law, KVL) The algebraic sum of all voltages taken around a closed path in a circuit is zero.



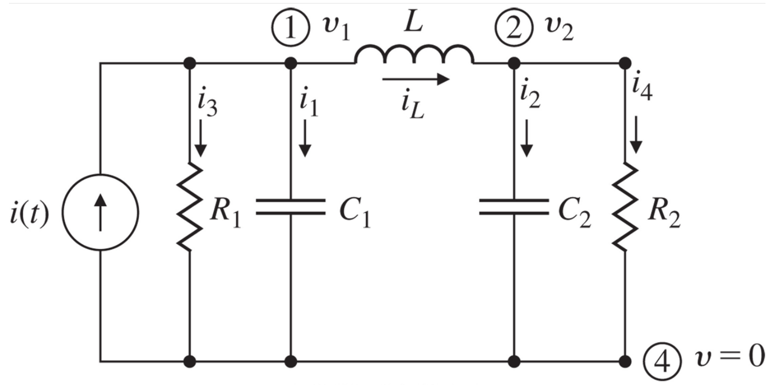
4. (Example 2.9, Bridged Tee Circuit) After assigning four node voltages, let us consider $v_4 = 0$ as reference voltage, then we know $v_1 = v_i$ and $v_3 = v_o$ at nodes 1 and 3, respectively, and, at nodes 2 and 3, we have

$$\frac{v_2 - v_1}{R_1} + C_1 \frac{dv_2}{dt} + \frac{v_2 - v_3}{R_2} = 0 \qquad \frac{v_3 - v_2}{R_2} + C_2 \frac{d(v_3 - v_1)}{dt} = 0$$

Let us take LT to derive TF $\frac{V_o(s)}{V_i(s)}$:

$$\left(\frac{1}{R_1} + C_1 s + \frac{1}{R_2}\right) V_2(s) = \frac{1}{R_1} V_i(s) + \frac{1}{R_2} V_o(s) \quad \rightarrow \quad (R_1 + R_2 + R_1 R_2 C_1 s) V_2(s) = R_2 V_i(s) + R_1 V_o(s)$$

$$\left(\frac{1}{R_2} + C_2 s\right) V_o(s) = \frac{1}{R_2} V_2(s) + C_2 s V_i(s) \quad \rightarrow \quad (1 + R_2 C_2 s) V_o(s) = V_2(s) + R_2 C_2 s V_i(s)$$



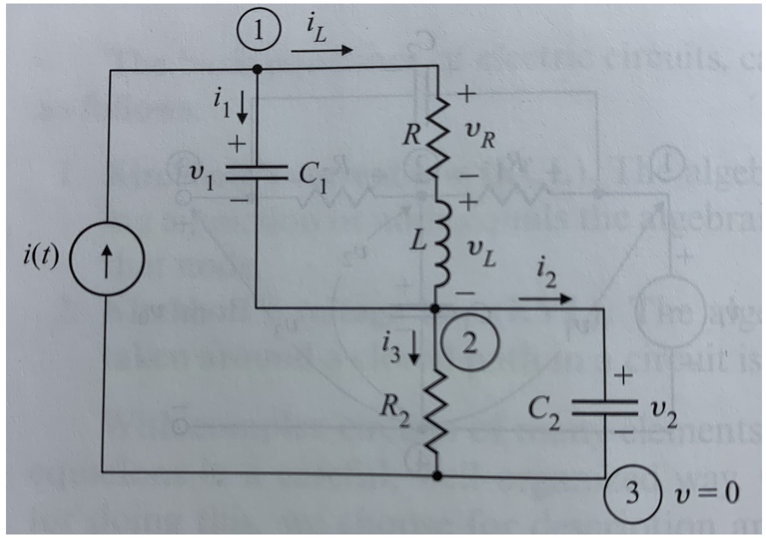
5. (Example, Circuit with Current Source)

$$-i(t) + \frac{v_1}{R_1} + C_1\frac{dv_1}{dt} + i_L = 0 \quad \rightarrow \quad I(s) = \left(\frac{1}{R_1} + C_1 s\right) V_1(s) + I_L(s)$$

$$-i_L + C_2\frac{dv_2}{dt} + \frac{v_2}{R_2} = 0 \quad \rightarrow \quad I_L(s) = \left(C_2 s + \frac{1}{R_2}\right) V_2(s)$$

$$v_1 - v_2 = L\frac{di_L}{dt} \quad \rightarrow \quad V_1(s) - V_2(s) = LsI_L(s)$$

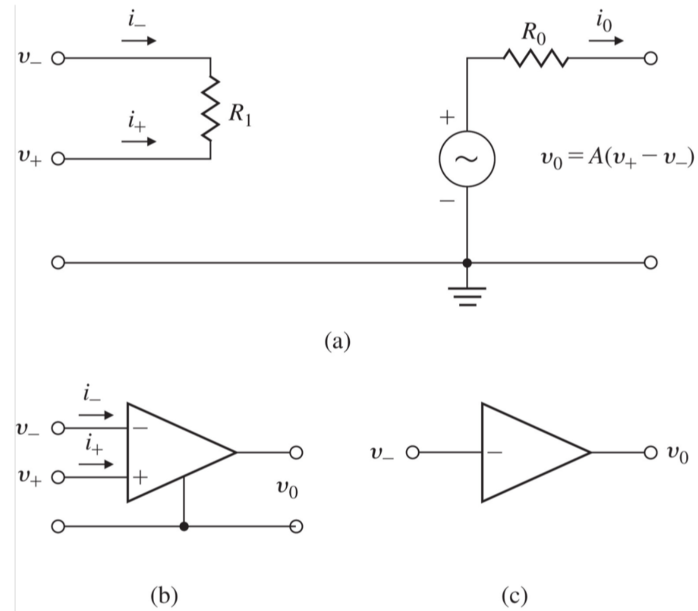
After tedious manipulations, we can derive two TF's such as $\frac{V_1(s)}{I(s)}$ and $\frac{V_2(s)}{I(s)}$



6. (Example 2.10, Circuit with Current Source)

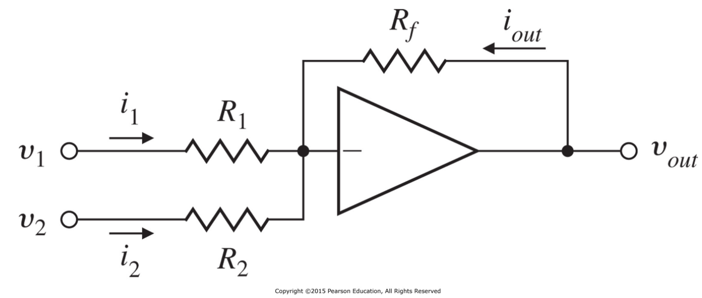
$$C_1 \frac{dv_1}{dt} + i_L = i(t) \quad \rightarrow \quad C_1 s V_1(s) = I(s) - I_L(s)$$
$$R i_L + L\frac{di_L}{dt} = v_1 \quad \rightarrow \quad (Ls + R) I_L(s) = V_1(s)$$
$$\frac{v_2}{R_2} + C_2 \frac{dv_2}{dt} = i(t) \quad \rightarrow \quad (C_2 s + 1/R_2) V_2(s) = I(s)$$

After tedious manipulations, we can derive three TF's such as $\frac{V_1(s)}{I(s)}$, $\frac{V_2(s)}{I(s)}$ and $\frac{I_L(s)}{I(s)}$



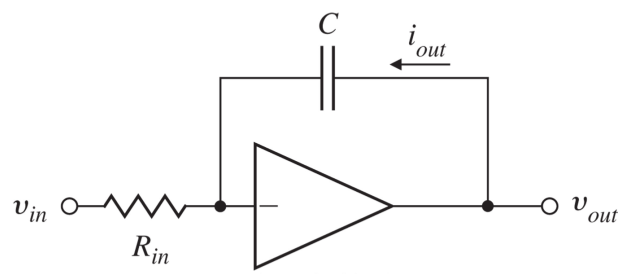
7. Consider the operational amplifier (OP-amp), for a practical use in control circuits, assume that the op-amp is ideal with the

$R_1 = \infty$ $\quad R_0 = 0$ $\quad A = \infty$ $\quad i_+ = i_- = 0$ $\quad v_+ = v_-$



8. (Example 2.11, Op-Amp Summing Circuit) weighted sum of the input voltages with a sign change.

$$\frac{0-v_1}{R_1}+\frac{0-v_2}{R_2}+\frac{0-v_o}{R_f}=0 \qquad \rightarrow \qquad v_o=-\left(\frac{R_f}{R_1}v_1+\frac{R_f}{R_2}v_2\right)$$



9. (Example 2.12, Op-Amp Integrator Circuit) integration of the input voltage with a sign change.

$$\frac{0-v_i}{R_i}+C\frac{d(0-v_o)}{dt}=0 \qquad \rightarrow \qquad v_o(t)=-\frac{1}{R_iC}\int_0^t v_i(\tau)d\tau+v_o(0)$$

The TF with zero initial condition, we have

$$\frac{V_o(s)}{V_i(s)}=\frac{1}{R_iCs}$$