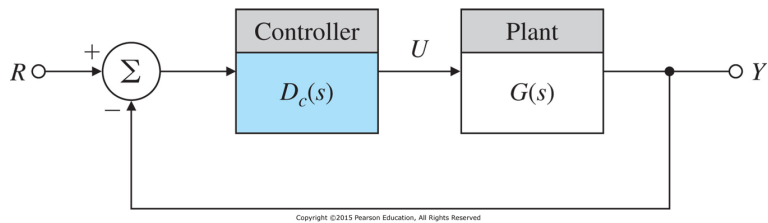


4 Design Using Dynamic Compensation

- Lead compensation approximates the function of PD control and acts mainly to speed up a response by lowering “rise time” and decreasing the “transient overshoot”.
- Lag compensation approximates the function of PI control and is usually used to improve the “steady-state accuracy” of the system
- Notch compensation will be used to achieve stability for systems with lightly damped flexible modes, as we saw with the satellite attitude control having non-collocated actuator and sensor.
- Compensation with a TF of the form:

$$D_c(s) = K \frac{s + z}{s + p}$$

- is called “lead” compensation if $z < p$
- “lag” compensation if $z > p$

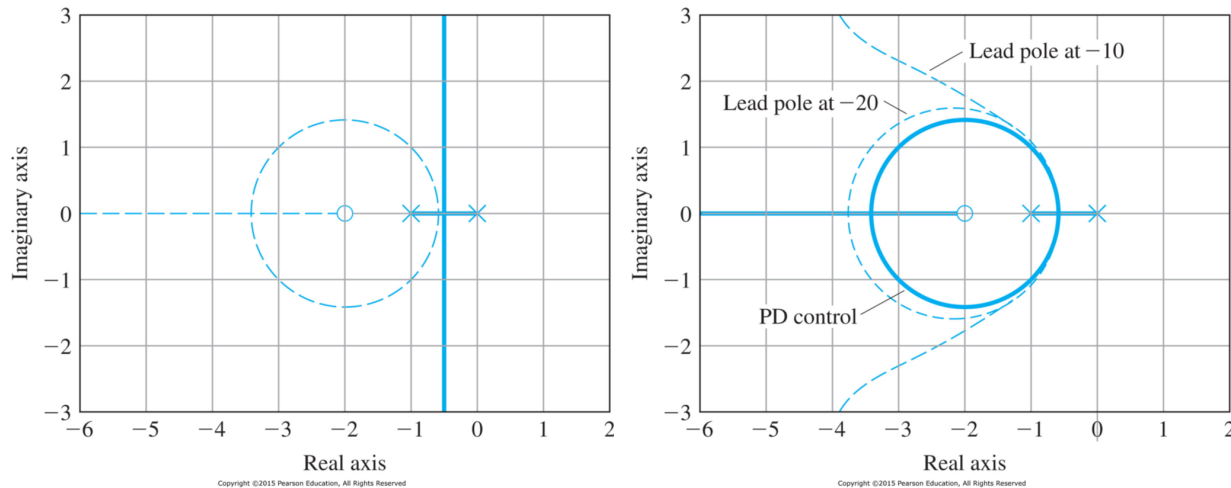


- The characteristic equation of the system in Fig. 5.21 is

$$1 + D_c(s)G(s) = 0 \quad \rightarrow \quad 1 + KL(s) = 0$$

For further explanations, we assume $G(s) = \frac{1}{s(s+1)}$

1. Design using Lead Compensations



- **P-Control:** If $D_c(s) = K$, the root locus w.r.t. K is shown as the solid-line portion of the locus in Fig. 5.22
- **PD-Control:** If $D_c(s) = K(s+2)$, the modified root locus is the dashed-line of the locus in Fig. 5.22

- Pure derivative control is not normally practical because of the amplification of sensor noise implied by the differentiation.
- Lead compensation

$$D_c(s) = K \frac{s + 2}{s + p} \quad \text{with } p > 2$$

- Effect of the pole is to lower the damping, but for the early part of the locus, the effect of the pole is not great if $p > 20$ (See the Fig. 5.23)
- If the pole is too close to the zero, the root locus moves back too far toward its uncompensated shape and the zero is not successful in doing its job. (See the Fig. 5.23)
- When the pole is too far to the left, the magnification of sensor noise appearing at the output of $D_c(s)$ is too great.
- Normally, the pole of the Lead Compensator is located at a distance 5 to 20 times the value of the zero location.
- The quantity of phase lead is given by

$$\phi = \tan^{-1} \frac{\omega}{z} - \tan^{-1} \frac{\omega}{p}$$

since $z < p$, $\phi > 0$ indicates the quantity of phase lead.

- (Example 5.11, Design using Lead Compensation) Find the Lead Compensator $D_c(s) = K \frac{s+2}{s+p}$ that will provide $M_p \leq 0.2$ and $t_r \leq 0.3$

a) From the requirements,

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \leq 0.2 \quad \rightarrow \quad \zeta \geq 0.46$$

$$t_r = \frac{1.8}{\omega_n} \leq 0.3 \quad \rightarrow \quad \omega_n \geq 6$$

To provide some margin, let us choose $\zeta \geq 0.5$ and $\omega_n \geq 7$

b) Try $D_c(s) = K \frac{s+2}{s+10}$.

i. (Rule 1, Start and End)

when $K = 0$, $s = 0, -1, -10$ poles of $L(s)$

when $K = \infty$, $s = -2, \infty, \infty$ zeros of $L(s)$

ii. (Rule 2, Real Axis) Negative real parts of $-10 \leq s \leq -2$ and $-1 \leq s \leq 0$ are loci.

iii. (Rule 3, Asymptotes)

$$\phi_l = \frac{180^\circ + 360(l-1)}{2} = \pm 90^\circ$$

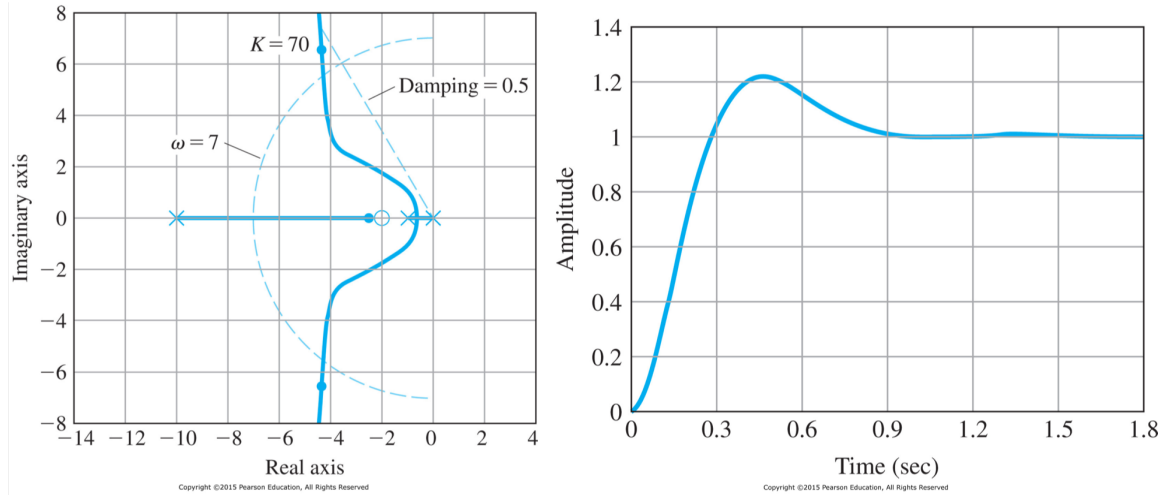
$$\alpha = \frac{(-11) - (-2)}{2} = -4.5$$

iv. (Rule 4, Departure Angles and Arrival Angles) They are clear.

v. (Rule 5, Break-in and Breakaway Points)

$$\frac{dK}{ds} = -\frac{d}{ds} \frac{s(s+1)(s+10)}{s+2} = \frac{2s^3 + 17s^2 + 44s + 20}{(s+2)^2} = 0 \quad \rightarrow \quad s = -0.5727 \quad \text{breakaway}$$

vi. Fig. 5.24 shows that $K = 70$ will yield $\zeta = 0.56$ and $\omega_n = 7.7$. The time step-response will be Fig. 5.25.



c) Try MATLAB (Fig. 5.26)

```
s = tf('s');
sysG = 1/(s*(s+1));
sysD = (s+2)/(s+10);
rltool(sysG,sysD)
```

2. Design using Lag Compensations

- In order to increase the velocity error constant K_v for the system type 1, the lag compensator is required.
- Lag compensator has

$$D_c(s) = \frac{s + z}{s + p} \quad z > p$$

where the values of z and p are small compared with ω_n .

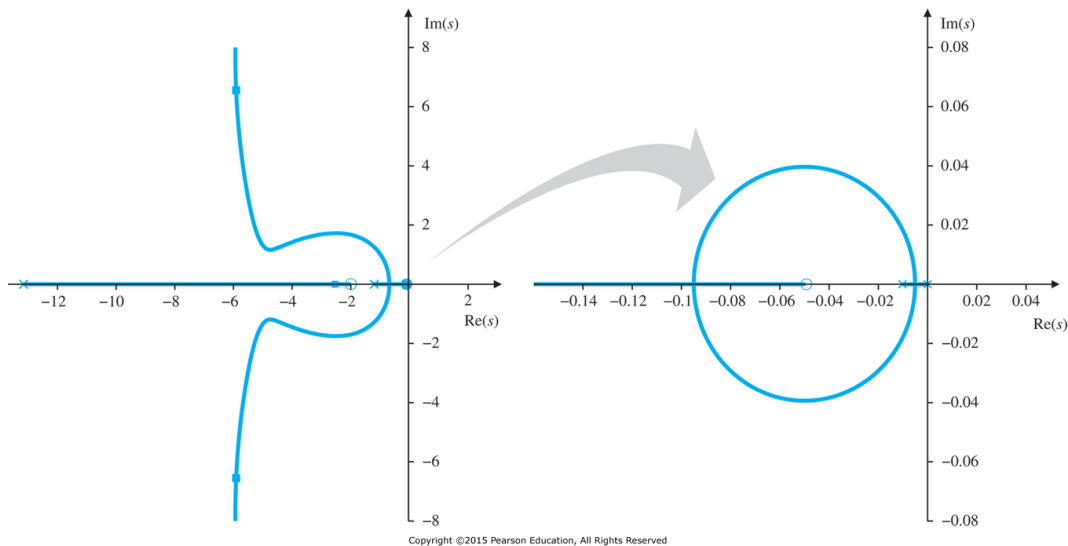
- Because $z > p$, the phase $\phi = \tan^{-1} \frac{\omega}{z} - \tan^{-1} \frac{\omega}{p}$ is negative, corresponding to phase lag.
- Consider $G(s) = \frac{1}{s(s+1)}$ with lead compensation $KD_{c1}(s) = 91 \frac{s+2}{s+13}$ that produced the locus in Fig. 5.26, we have the velocity error constant as

$$\begin{aligned} K_v &= \lim_{s \rightarrow 0} sKD_{c1}(s)G(s) \\ &= \lim_{s \rightarrow 0} s \times 91 \times \frac{s+2}{s+13} \frac{1}{s(s+1)} \\ &= 14 \end{aligned}$$

- Suppose we require that $K_v = 70$ in order to reduce the velocity error by a factor of 5. Thus we need $\frac{z}{p} = 5$ near the origin, the pole and zero of lag compensator are determined as

$$p = -0.01 \quad \text{and} \quad z = -0.05 \quad \rightarrow \quad D_{c2}(s) = \frac{s + 0.05}{s + 0.01}$$

- The resultant root locus is changed into Fig. 5.28. The effect of the lag compensation can be seen by expanding the region of the locus around the origin, as shown on the right side of Fig. 5.28



3. Design using Notch Compensations

- Suppose the design has been completed with lead and lag compensation given by

$$KD_c(s) = 90 \frac{s + 2}{s + 13} \frac{s + 0.05}{s + 0.01}$$

- Again, suppose the system has a substantial oscillation about 50rad/s when tested, then the plant has the flexibility effect as follow:

$$G(s) = \frac{2500}{s(s + 1)(s^2 + s + 2500)}$$

where the root locus is similar to Fig. 5.18 with very lightly damped roots at 50rad/s .

- Two possibilities to reduce the flexibility effect,
 - Gain stabilization: reducing the gain at the high frequency. Sometimes, the response time resulting from gain stabilization could be too long.

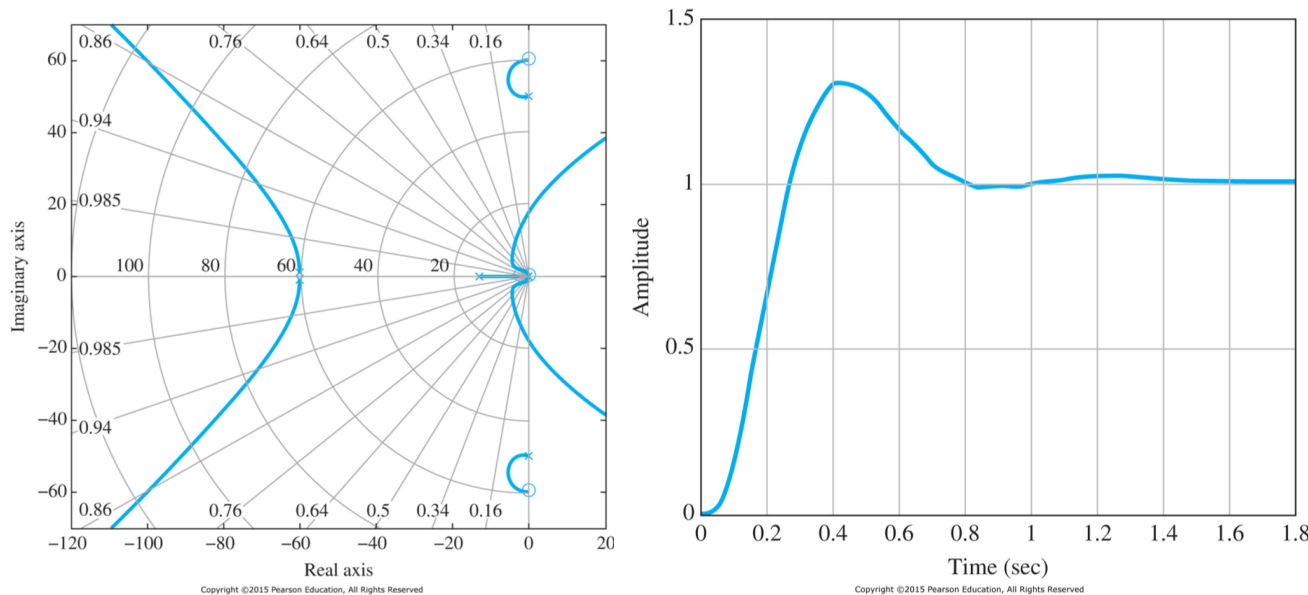
– Phase stabilization: adding a zero near the resonance so as to shift the departure angles from the resonant poles so as to cause the closed-loop root to move into the LHP, thus causing the associated transient to die out.

- For phase stabilization, let us apply the notch compensation as follow:

$$D_{notch}(s) = \frac{s^2 + 2\zeta\omega_o s + \omega_o^2}{s^2 + 2\omega_o s + \omega_o^2} = \frac{s^2 + 0.8s + 3600}{s^2 + 120s + 3600} \rightarrow \omega_o = 60 \quad \zeta = 0.0067$$

where it is necessary to place the frequency of the notch above that of the resonance to get the departure angle to point toward the LHP.

- New root locus is shown in Fig. 5.29 and step response is shown in Fig. 5.30.



4. Analog and Digital Implementations

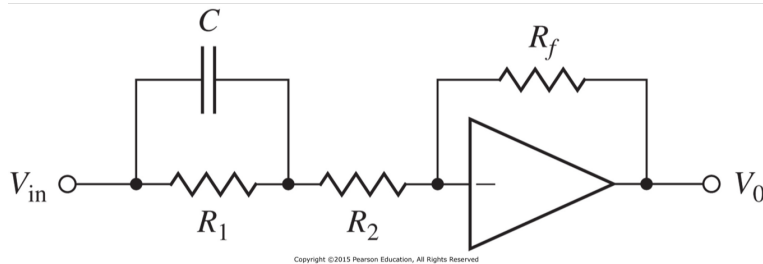
- Most compensators can be implemented using analog electronics and using digital devices.
- For example, the lead compensator shown in Fig. 5.31 (analog implementation) can be realized as

$$\frac{0 - V_{in}}{R_2 + R_1 \parallel \frac{1}{Cs}} + \frac{0 - V_o}{R_f} = 0 \quad \rightarrow \quad \frac{V_o}{R_f} = -\frac{V_{in}}{R_2 + \frac{R_1}{1+R_1Cs}}$$

$$V_o = -\frac{R_f + R_f R_1 Cs}{(R_1 + R_2) + R_1 R_2 Cs} V_{in}$$

If $R_f = R_1 + R_2$, then

$$D_{lead}(s) = -a \frac{s + z}{s + p} \quad \text{with} \quad a = \frac{R_1 + R_2}{R_2} \quad z = \frac{1}{R_1 C} \quad p = \frac{R_1 + R_2}{R_1 R_2 C}$$



5 Design Examples Using the Root Locus

6 Extensions of the Root-Locus Method

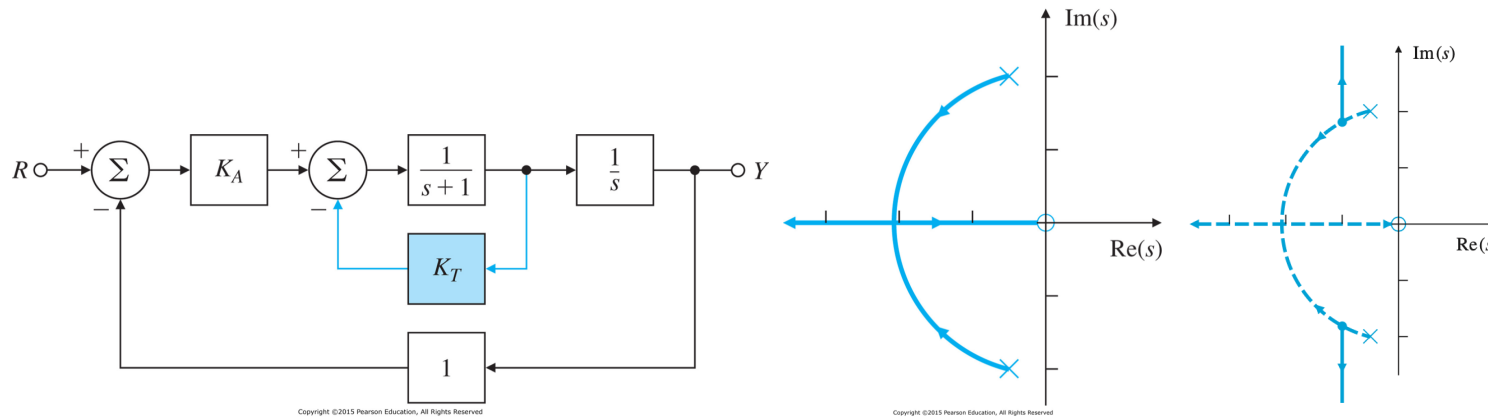
- The root locus technique is a graphical scheme to show locations of possible roots of an algebraic equation as a single real parameter varies.
- The method can be extended to consider negative values of the parameter and a sequential consideration of more than one parameter.

1. Rules for Plotting Negative Root Locus (skipped)

2. Successive Loop Closure

- An important technique for practical control is to consider a structure with two loops, an inner loop around an actuator or part of the process dynamics and an outer loop around the entire plant-plus-inner-controller.
 - the inner loop controller is chosen to be robust and to give good response alone.
 - the outer loop can be designed to be simpler and more effective than if the entire control was done without the aid of the inner loop.

- (Example 5.15. Root Locus Using Two Parameters in Succession) Consider the Fig. 5.44



a) The characteristic equation:

$$1 + \frac{K_A}{s(s+1)} + \frac{K_T}{s+1} = 0 \quad \rightarrow \quad s^2 + s + K_A + K_T s = 0$$

b) Assume $K_A = 4$, the characteristic equation is

$$1 + K_T L(s) = 0 \quad \text{with} \quad L(s) = \frac{s}{s^2 + s + 4}$$

For this locus, the zero is at $s = 0$ and the poles are at $s = -0.5 \pm 1.94j$. The sketch is shown in Fig. 5.45. Then we can choose $K_T = 1$.

c) For given $K_T = 1$, let us assign $K_A = 4 + K_1$ so that we can change K_A parameter for the better performance. Then the characteristic equation is

$$1 + K_1 L(s) = 0 \quad \text{with} \quad L(s) = \frac{1}{s^2 + 2s + 4}$$

It is noted that the poles of the new locus are the roots of the previous locus, $s = -1 \pm \sqrt{3}j$.