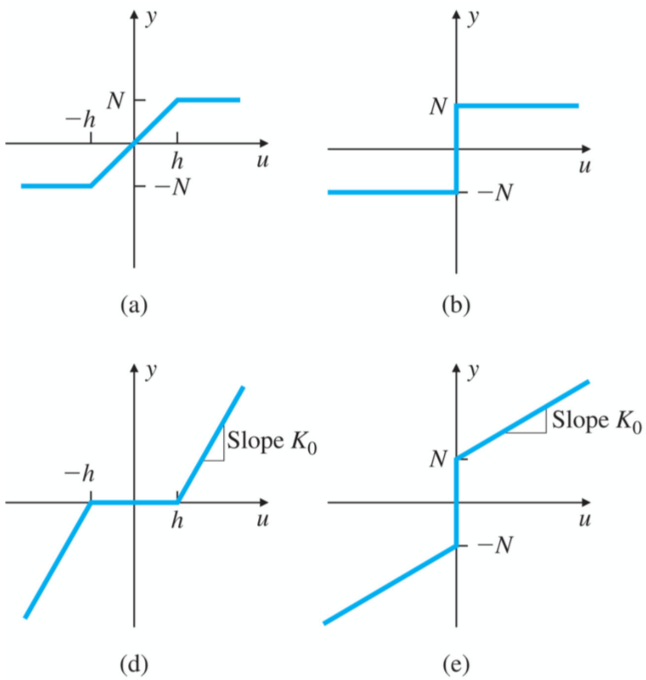
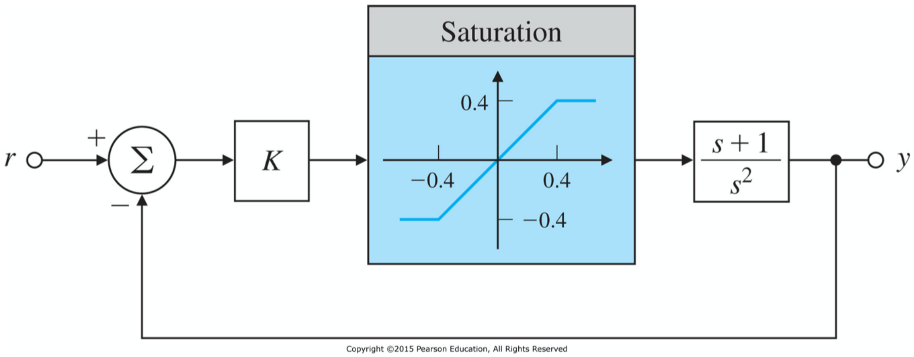
3 Equivalent Gain Analysis using the Root Locus

- Typical examples of memoryless nonlinearities are listed as *saturation, relay, relay with dead zone, gain with dead zone, preloaded spring, coulomb plus viscous friction, and quantization.*

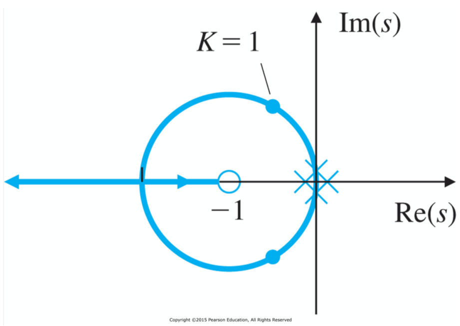


- For given memoryless nonlinearities, the technique is to replace the *memoryless nonlinearity* by an equivalent gain K, and a root locus is plotted versus this gain.

- (Example 9.6) Consider the system with saturation. Determine the stability properties of the system using the root-locus technique ?

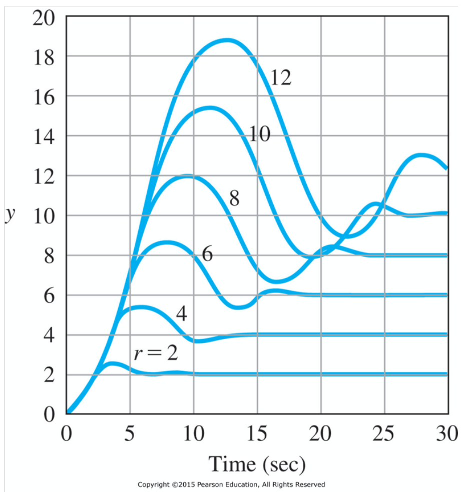


1. The root locus with saturation removed is given by the figure.

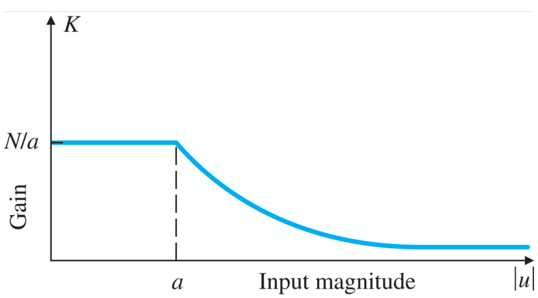


2. At $K = 1$, the damping ratio is $\zeta = 0.5$. As the gain is reduced, the locus shows that the roots move toward the origin of the s-plane with less and less damping.

3. For $r = 2, 4, 6, 8, 10, 12$, the step responses are shown in the following figure. As long as the signal entering the saturation remains less than 0.4, the system will be *linear* and should behave according to the roots at $\zeta = 0.5$. However, *as the input gets larger, the response has more and more overshoot and slower and slower recovery.*

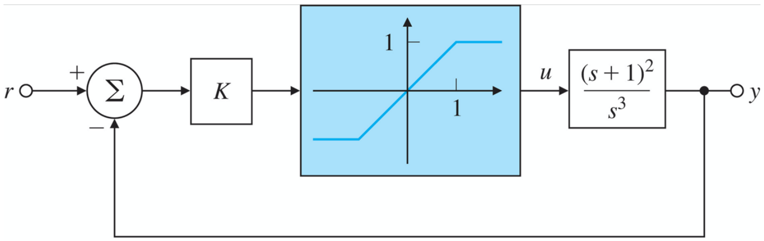


4. This can be explained that *larger and larger input signals correspond to smaller and smaller effective gain* K, as seen in Fig. 9.10

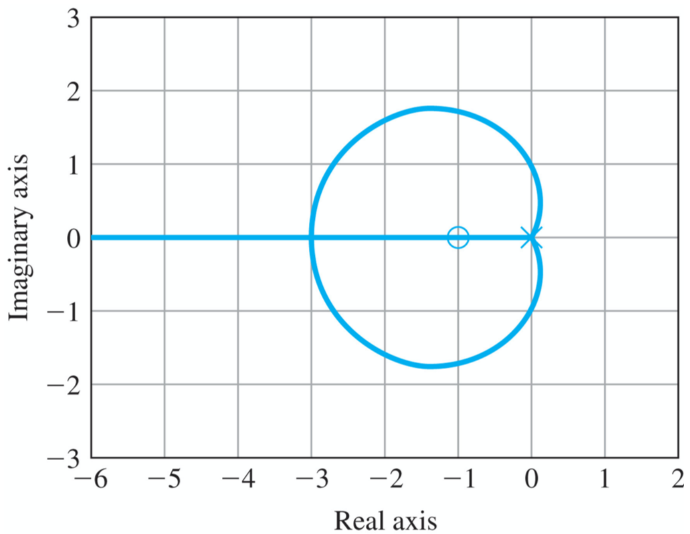


5. As K decreases, the closed-loop poles move closer to the origin and have a smaller damping ζ. This results in longer rise and settling times, increases overshoot, and greater oscillatory response.

- (Example 9.7) Consider the system with a saturation nonlinearity. Determine whether the system is stable?

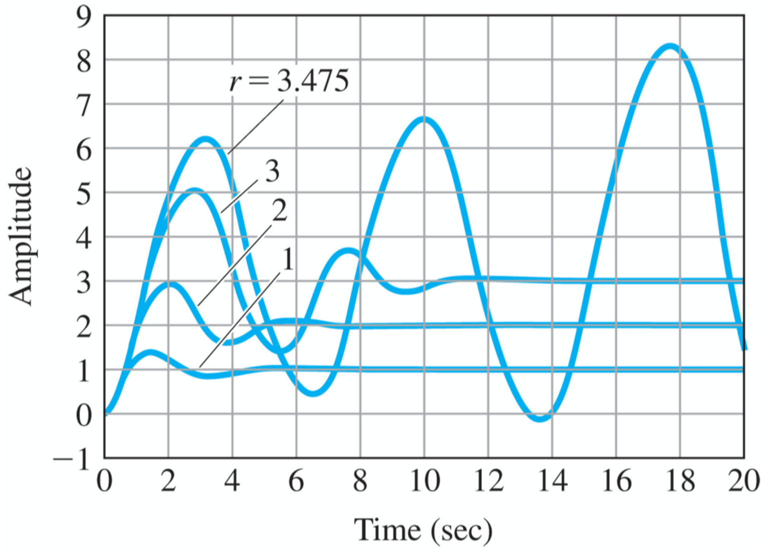


1. The root locus for the system, excluding the saturation, is plotted. Imaginary axis crossing occurs at $\omega_o = 1[rad/s]$ and $K = \frac{1}{2}$.
2. The system is stable for large gains but unstable for smaller gains.

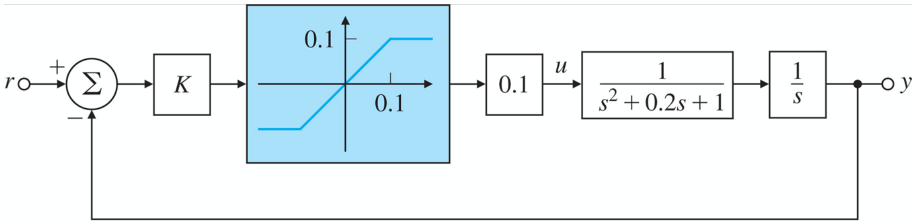


3. If $K = 2$, $\zeta = 0.5$ for smaller reference input signals. However, *as the reference input size gets larger, the equivalent gain would get smaller due to the saturation, and the system would be expected to become less well damped.* Finally the system would be expected to become unstable at some point for large inputs.

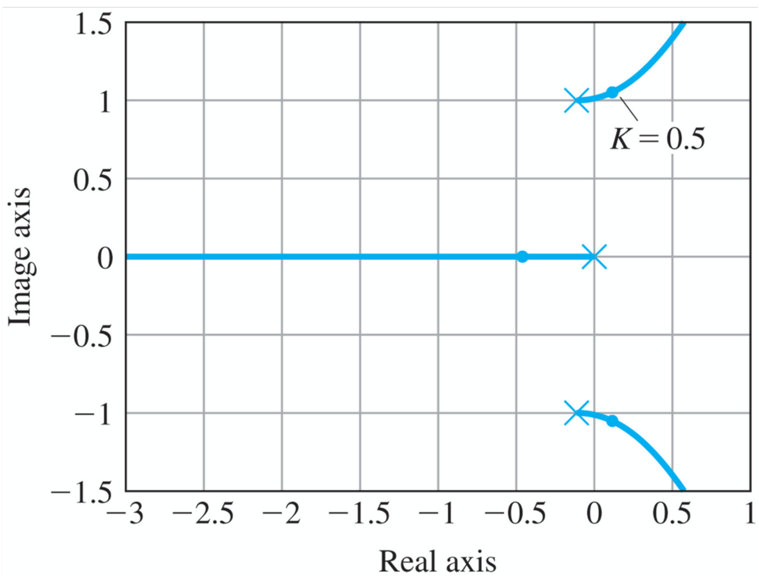
4. Step response with $K = 2$ for input steps of size $r = 1, 2, 3, 3.4$ are shown in Fig. 9.13.



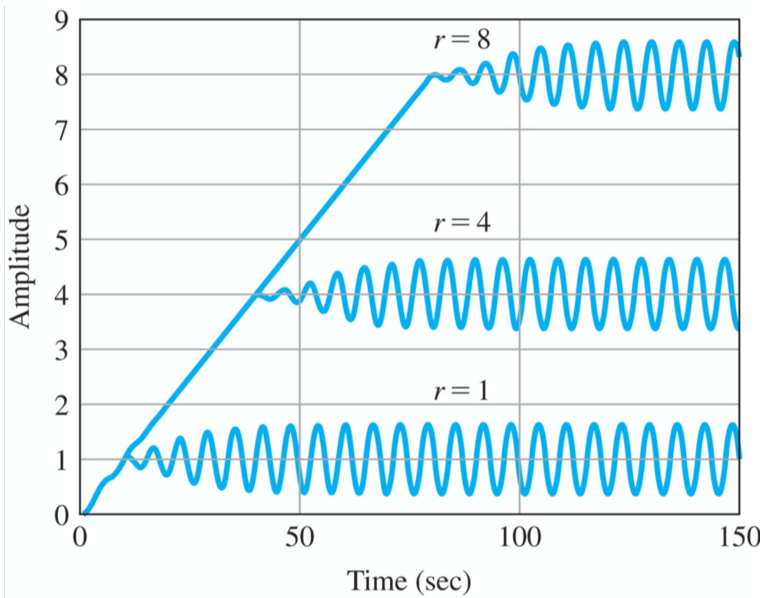
- (Example 9.8) Consider the system. Determine whether the system is stable and find the amplitude and frequency of the *limit cycle*. Modify the controller design to minimize the effect of limit-cycle oscillations.



1. The denominator term $s^2 + 0.2s + 1 = 0$ implies $\omega = 1$ and $\zeta = 0.1$. The root locus for this system versus K, excluding the saturation, is sketched.
2. The imaginary axis crossing can be verified to be at $\omega_0 = 1$, $K = 0.2$, thus a gain of $K = 0.5$ is enough to force the roots of the resonant mode into the RHP, as shown by the dots.



3. Plots of the step responses with $K = 0.5$ for three steps of size $r = 1, 4, 8$ are shown. Due to saturation effect, the effective gain is lowered to $K = 0.2$ from $K = 0.5$ and then stop growing!



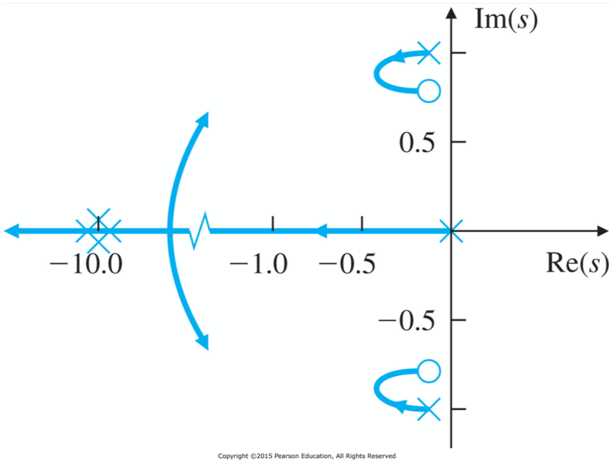
(a)

4. The error builds up to a *fixed amplitude* and then starts to oscillate at a *fixed amplitude*. The oscillations have a frequency of 1 [rad/s] and hold constant amplitude regardless of the step sizes of the input. $\rightarrow$ *limit cycle*

5. In order to prevent the limit cycle, the locus has to be modified by compensation so that no branches cross into the RHP. A remedy is to place compensation zeros near the lightly damped poles as shown in Fig. 9.17

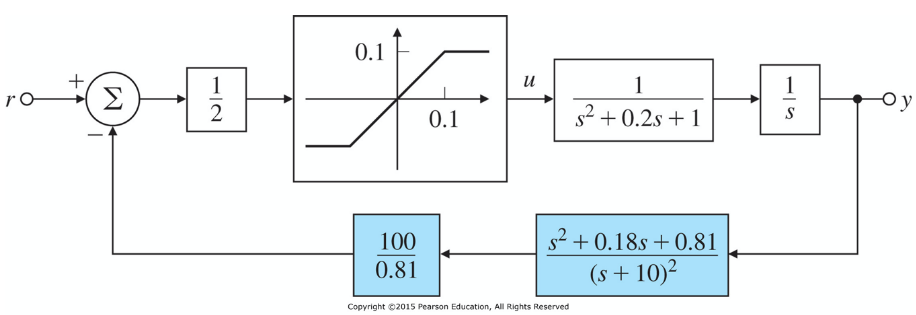
6. For example, let us design the compensator as follows:

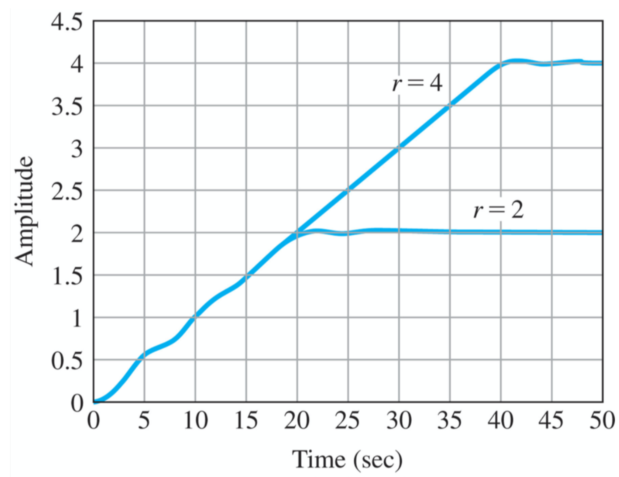
$$D_c(s) = 123\frac{s^2 + 0.18s + 0.81}{(s+10)^2}$$



where 123 has been selected to make the compensation's DC gain equal to unity.

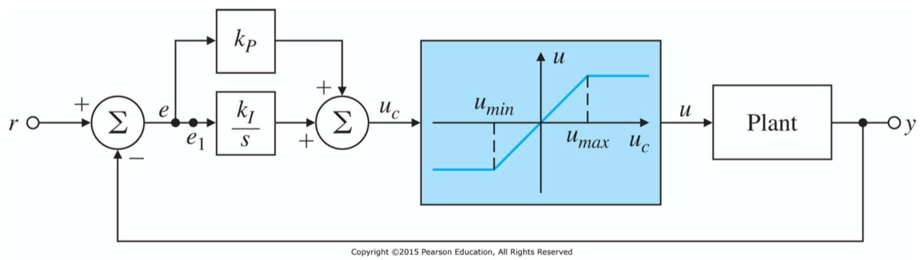
7. This notch compensation attenuates inputs in the vicinity of $\omega_n^2 = 0.81$ or $\omega_n = 0.9$, so that any input from the plant resonance is attenuated and is thus prevented from detracting from the stability of the system. (see Figs. 9.18 and 9.19)



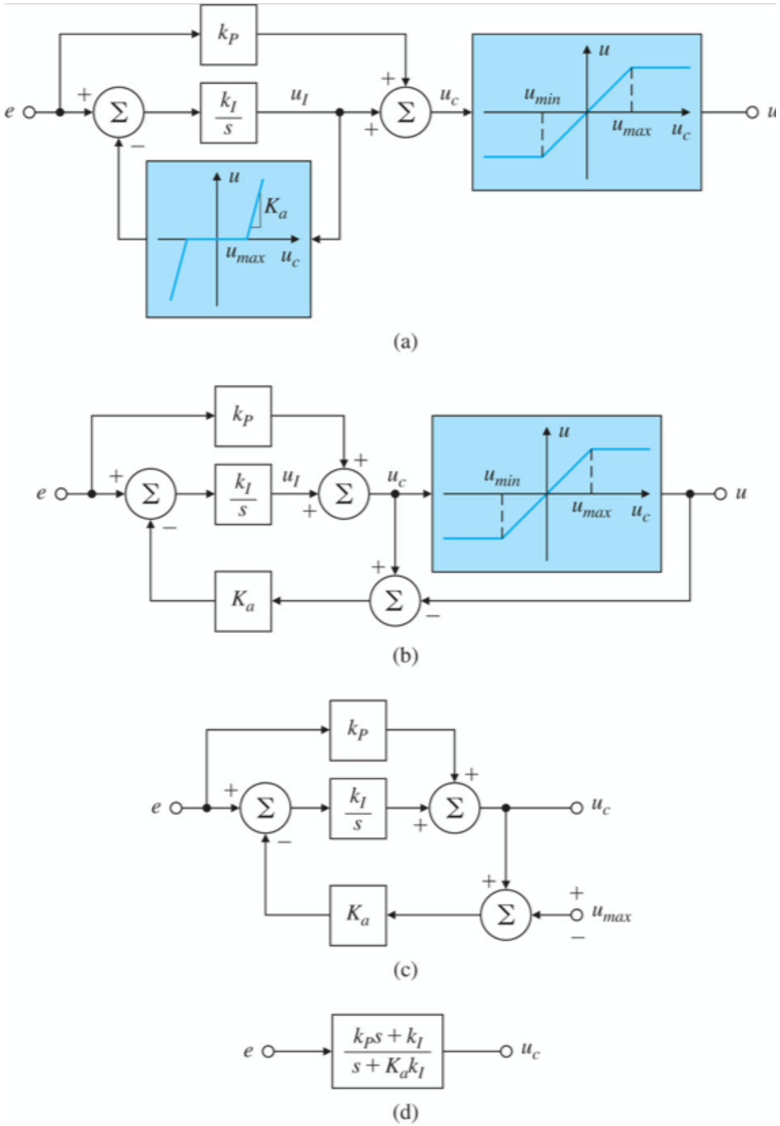


- (9.3.1) Integrator Antiwindup

1. Consider the feedback system. Suppose a given reference step is more than large enough to cause the actuator to saturate at u_{max}.



2. The integrator continues integrating the error e and the signal u_c keeps growing. However, the input to the plant is stuck at its maximum value, namely, $u = u_{max}$.
3. The solution to this problem is an *integrator antiwindup circuit*, which turns off the integral action when the actuator saturates.
4. The effect of the antiwindup is *to reduce both the overshoot and the control effort* in the feedback system.
5. Effect of the saturation is to open the feedback loop and leave the open-loop plant with a constant input and the controller as an open-loop system with the system error as input.
6. If the controller is implemented digitally, by including a statement such as "if $|u| = u_{max}$, $k_I = 0$. Fig. 9.21(a) is somewhat easier to understand. Fig. 9.21(b) is easier to implement. Fig. 9.21(c) shows block diagram. Fig. 9.21(d) : the integrator part becomes the first-order lag, where K_a is the *antiwindup gain* chosen to be large enough that the antiwindup circuit keeps the input to the integrator small under all error conditions.



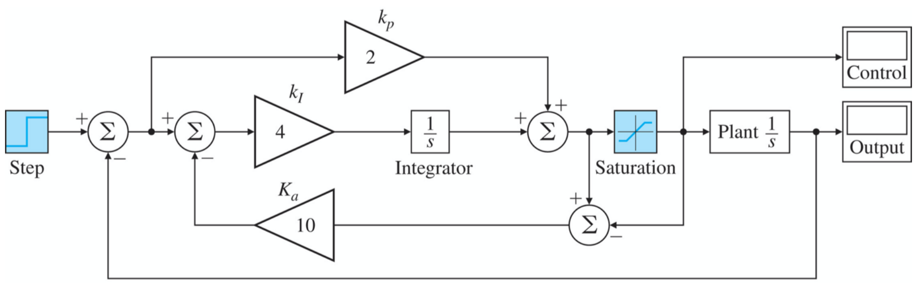
(a)
(b)
(c)
(d)

7. (Example 9.9) Consider a plant and a PI controller

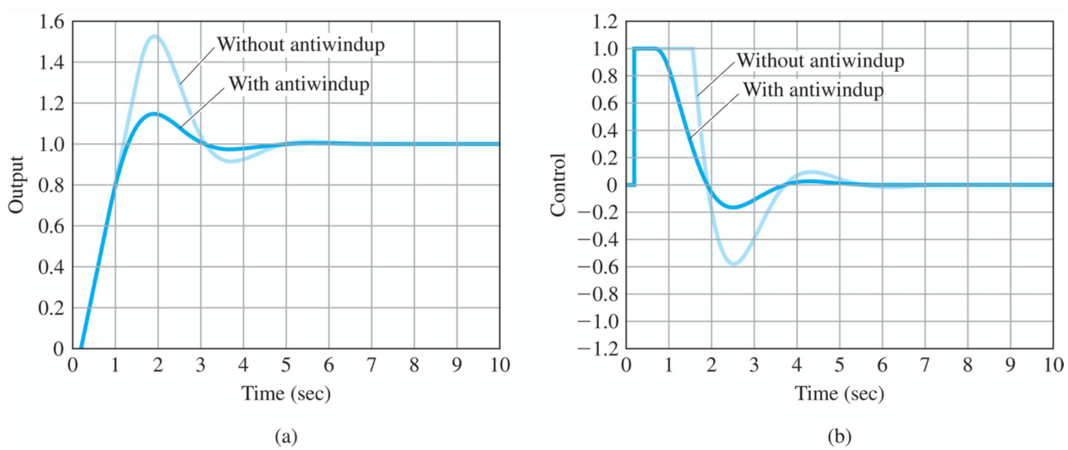
$$G(s) = \frac{1}{s} \qquad\qquad D_c(s) = k_P + \frac{k_I}{s} = 2 + \frac{4}{s}$$

in the unity feedback configuration. The input to the plant is limited to ± 1.

a) With $K_a = 10 > K_I = 4$, let us make simulation as shown in Fig. 9.22



b) Fig. 9.23 shows the step response and the corresponding control effort.



c) The system with antiwindup has substantially less overshot and less control effort.