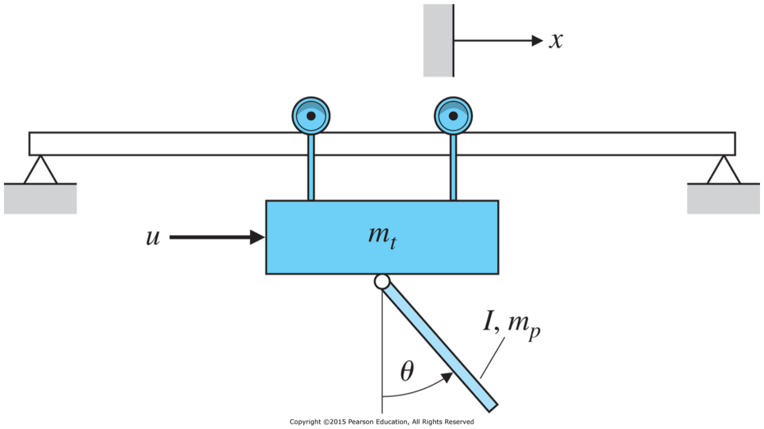
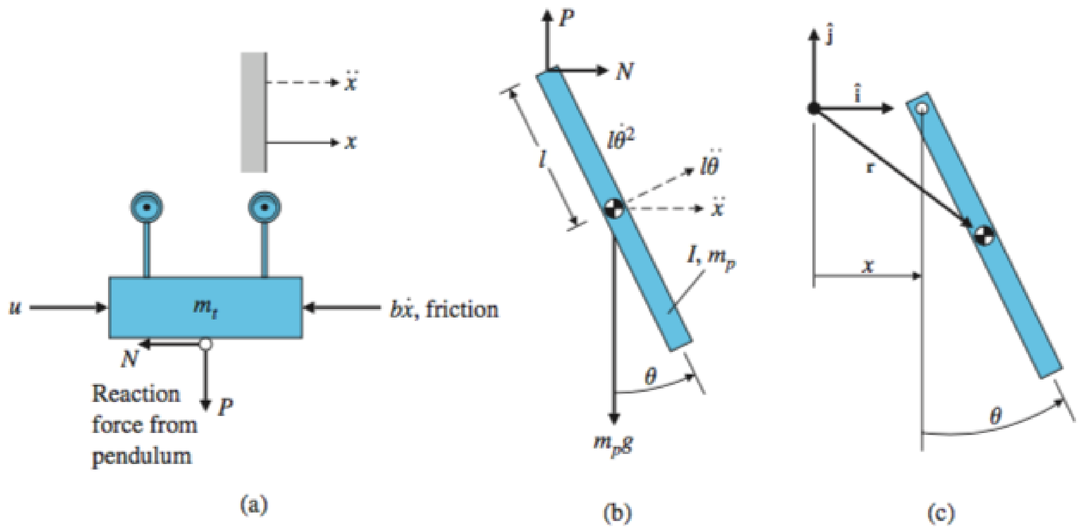
(Revisited Example 2.8)




For the pendulum,

$$N = m_p\ddot{x} + m_pl\ddot{\theta}\cos\theta - m_pl\dot{\theta}^2\sin\theta$$

$$P - m_pg = m_pl\ddot{\theta}\sin\theta + m_pl\dot{\theta}^2\cos\theta$$

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \qquad \rightarrow \qquad (I + m_pl^2)\ddot{\theta} + m_pgl\sin\theta = -m_p\ddot{x}l\cos\theta$$

For the crane,

$$m_t\ddot{x} = u - N - b\dot{x} \qquad \rightarrow \qquad (m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta}\cos\theta - m_p l\dot{\theta}^2\sin\theta = u$$

As a result, we have the complete equation of motion:

$$(I + m_p l^2)\ddot{\theta} + m_p gl\sin\theta + m_p\ddot{x}l\cos\theta = 0$$
$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta}\cos\theta - m_p l\dot{\theta}^2\sin\theta = u$$

For the linearization with small angle variation, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ and $\dot{\theta}^2 \approx 0$, we have

$$(I + m_p l^2)\ddot{\theta} + m_p gl\theta + m_p l\ddot{x} = 0$$
$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta} = u$$

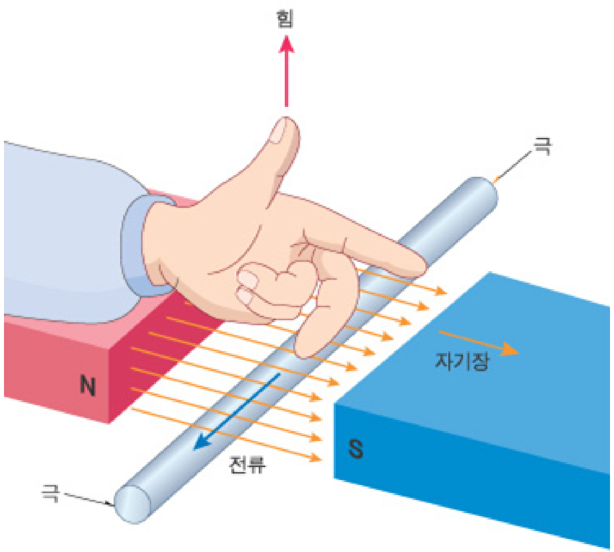
Furthermore, ignoring the damping $b$, we can get the TF as follow:

$$(I + m_p l^2)s^2\Theta(s) + m_p gl\Theta(s) + m_p ls^2X(s) = 0$$
$$(m_t + m_p)s^2X(s) + m_p ls^2\Theta(s) = u$$

Thus, we have

$$\therefore \qquad \frac{\Theta(s)}{U(s)} = \frac{-m_p l}{[(m_t + m_p)(I + m_p l^2) - m_p^2 l^2]s^2 + (m_t + m_p)m_p gl}$$

# 3 Models of Electromechanical Systems

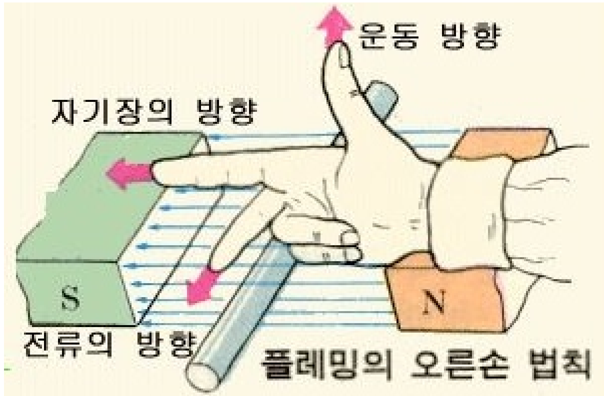



1. Law of Motor

   - If a current $i$[A] in a conductor of length $l$[m] is arranged at right angles in a magnetic field of $B$[Tesla], then there is a force on the conductor at right angles to the plane of $i$ and $B$ with magnitude

   $$F = Bil \ [N]$$

   - It is called "law of motors" regarding the conversion of electric energy into mechanical work.

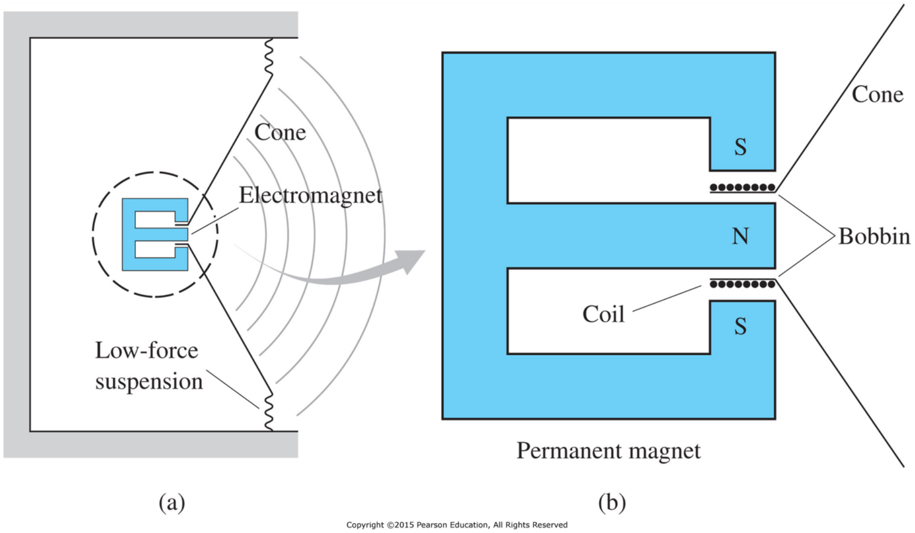



2. Law of Generator

   - If a conductor of length $l$[m] is moving in a magnetic field $B$[T] at a velocity of $v$[m/s] at mutually right angles, the electric voltage is established across the conductor with magnitude

$$e = Blv[V]$$

   - It is called "law of generators"




3. Loudspeakers
   (Example 2.13, Loudspeaker) A typical geometry for a loudspeaker for producing sound is sketched. The permanent magnet estabilshes a raidal field in the cylindrical gap b/w the poles of the magnet. The force on the conductor wound on the bobbin causes the voice coil to move, producing sound. The cone has mass $M$ and viscous friction $b$. Assume the magnet establishes a uniform field $B$ of 0.4[T] and the bobbin has 18 turns at a 1.9-cm diameter.

   - The conductor length is

$$l = (2 \cdot \pi \cdot 0.0095) \cdot 18 = 1.074[m]$$

   - The force is

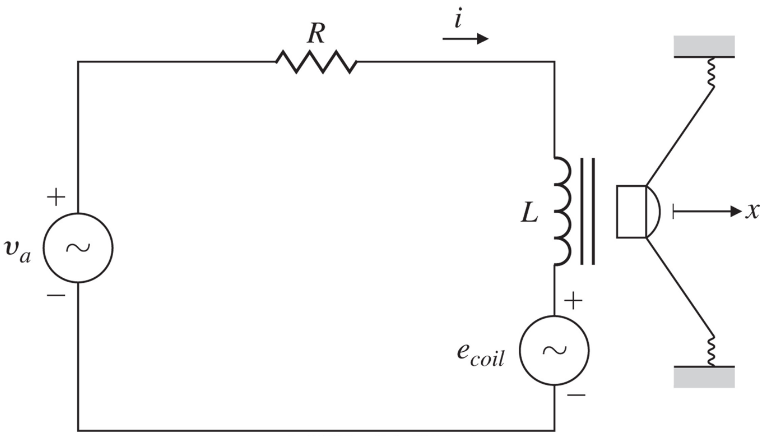
$$F = Bil = 0.4 \cdot i \cdot 1.074 = 0.43i[N]$$

- Mechanical system is modeled as

$$M\ddot{x} = F - b\dot{x} \qquad \rightarrow \qquad M\ddot{x} + b\dot{x} = 0.43i$$

- The TF of mechanical part is

$$\therefore \qquad \frac{X(s)}{I(s)} = \frac{0.43}{s(Ms+b)}$$





(Example 2.14, Loudspeaker with Circuit) Consider the driving circuit for the louspeaker. Find the differential equation relating the input voltage $v_a$ and the output cone displacement $x$. Assume the effective resistance $R$ and inductance $L$.

- The resulting voltage according to the speaker motion is

$$e_{coil} = Bl\dot{x} = 0.4 \cdot 1.047 \cdot \dot{x} = 0.43\dot{x}$$

- Due to the induced voltage effect, the electric circuit is modeled as

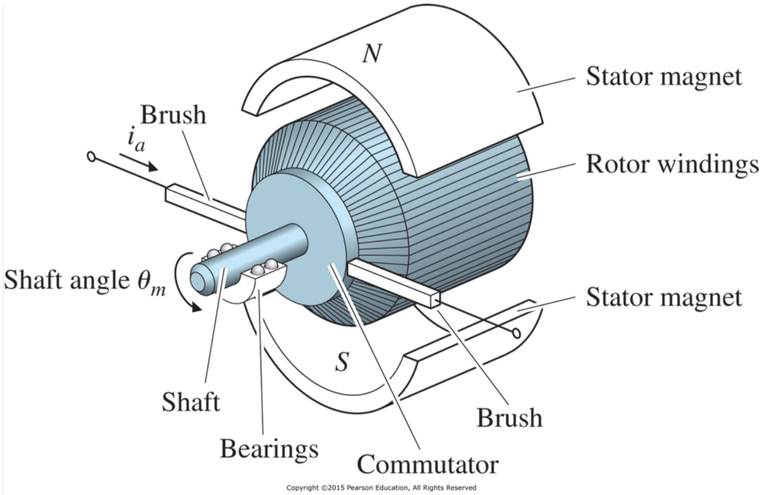
$$L\frac{di}{dt} + Ri + 0.43\dot{x} = v_a$$

$$(Ls + R)I(s) + 0.43sX(s) = V_a(s)$$

$$(Ls + R)\left(\frac{s(Ms + b)}{0.43}\right)X(s) + 0.43sX(s) = V_a(s)$$

- The TF of dynamic model for the loudspeaker is

$$\therefore \qquad \frac{X(s)}{V_a(s)} = \frac{0.43}{s[(Ms + b)(Ls + R) + 0.43^2]}$$

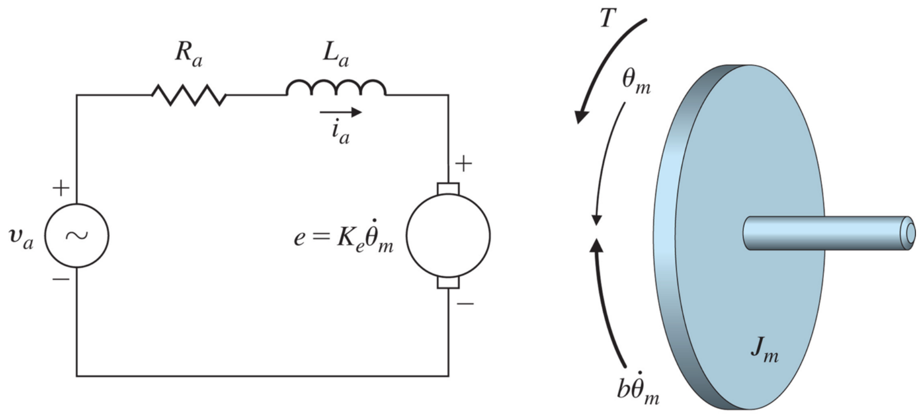



4. Motors

- Consider DC motor. The motor equations give the torque $T$ on the rotor in terms of armature current $i_a$ and express the back emf voltage in terms of shaft's rotational velocity $\dot{\theta}_m$.

$$T(t) = r \times (B \cdot i_a(t) \cdot l) = (rBl) \cdot i_a(t) = K_t \cdot i_a(t)$$
$$e(t) = B \cdot l \cdot (r \times \dot{\theta}_m(t)) = (rBl) \cdot \dot{\theta}_m(t) = K_e \cdot \dot{\theta}_m(t)$$

where $r$ implies an effective moment arm of motor, $K_t$ and $K_e$ denote the motor torque constant and the motor back emf constant. Note that $K_t = K_e$ with different dimensions.




(a) (b)



(Example 2.15, Modeling a DC Motor) Assume the rotor has inertia $J_m$ and viscous friction coefficient $b$.

$$J_m\ddot{\theta}_m + b\dot{\theta}_m = K_t i_a$$

$$L_a\frac{di_a}{dt} + R_a i_a + K_e\dot{\theta}_m = v_a$$

- Take LT with zero initial conditions, then we have

$$(J_m s^2 + bs)\Theta_m(s) = K_t I_a(s)$$

$$(L_a s + R_a)I_a + K_e s\Theta_m(s) = V_a(s)$$

- Also, we can get the TF as follow:

$$\therefore \quad \frac{\Theta_m(s)}{V_a(s)} = \frac{K_t}{s[(J_m s + b)(L_a s + R_a) + K_t K_e]}$$

- Ignoring the inductance due to small quantity, we can simplify above complete model into

$$\therefore \quad \frac{\Theta_m(s)}{V_a(s)} \approx \frac{K_t}{s[J_m R_a s + (bR_a + K_t K_e)]}$$
$$\approx \frac{K}{s(\tau s + 1)}$$

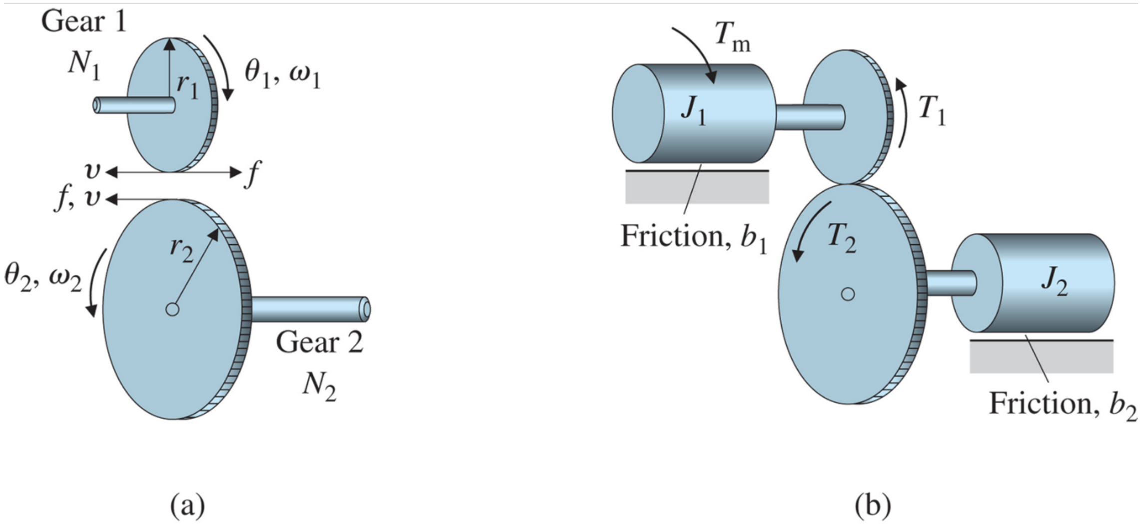
where

$$K = \frac{K_t}{bR_a + K_t K_e} \qquad \tau = \frac{J_m R_a}{bR_a + K_t K_e}$$

- If we consider the output speed, $\omega_m(t) = \dot{\theta}_m(t)$, then

$$\therefore \quad \frac{\Omega_m(s)}{V_a(s)} \approx \frac{K}{\tau s + 1}$$

- Other types of motors: AC motor, brushless DC motor, stepping motor and so on.





5. Gears

- Consider gear transmission. Since the transmitted force and velocity at the contact point are the same, we have

$$\frac{T_1}{r_1} = \frac{T_2}{r_2} = f : \quad \text{force applied by teeth at the contact point}$$

$$\omega_1 r_1 = \omega_2 r_2 = v : \quad \text{velocity at the contact point}$$

$$\frac{2\pi r_1}{N_1} = \frac{2\pi r_2}{N_2} = m : \quad \text{module for the transmission}$$

- Let us define the gear ratio $n = \frac{N_2}{N_1}$, then we have

$$n = \frac{N_2}{N_1} = \frac{r_2}{r_1} = \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

- The equations of motion for bodies 1 and 2 are

$$J_1\ddot{\theta}_1 + b_1\dot{\theta}_1 = T_m - T_1 \quad \rightarrow \quad nJ_1\ddot{\theta}_2 + nb_1\dot{\theta}_2 = T_m - \frac{T_2}{n}$$

$$J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 = T_2 \quad \rightarrow \quad J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 = nT_m - n^2J_1\ddot{\theta}_2 - n^2b_1\dot{\theta}_2$$

$$(J_2 + n^2J_1)\ddot{\theta}_2 + (b_2 + n^2b_1)\dot{\theta}_2 = nT_m$$

  where $T_m$ is a servo motor torque, $T_1$ is the reaction torque from gear 2 acting back on gear 1, and $T_2$ is the torque applied on gear 2 by gear 1.

- Take LT with zero initial conditions

$$\frac{\Theta_2(s)}{T_m(s)} = \frac{n}{J_{eq}s^2 + b_{eq}s}$$

  where

$$J_{eq} = J_2 + n^2J_1 \qquad b_{eq} = b_2 + n^2b_1$$

  are referred to as the equivalent inertias and damping coefficients.