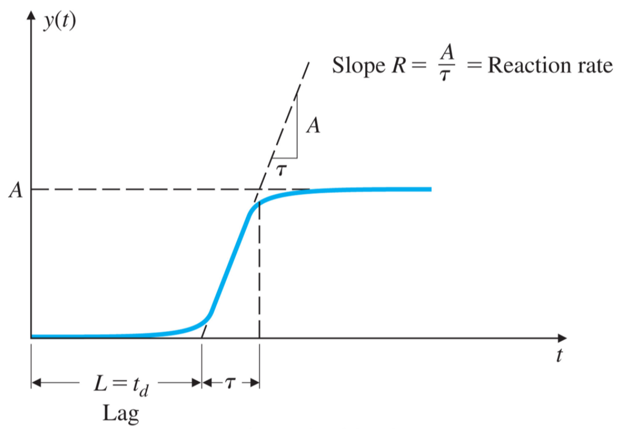
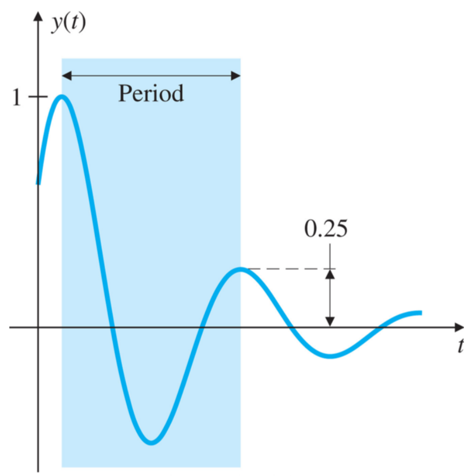
6. Ziegler-Nichols Tuning of the PID Controller

- History of PID control: Callender (1936), Ziegler and Nichols (1942, 1943)
- First method using step input (Quarter Decay Ratio Method)





  - When the step input is applied, if we can get S-shaped response curve (process reaction curve) shown in Fig. 4.18, then the first method is effective.
  - From the process reaction curve, the system can be modeled as follows:

$$\frac{Y(s)}{U(s)} = \frac{Ae^{-st_d}}{\tau s + 1}$$

  - With the aim of 25% decay ratio in one period shown in Fig. 4.19, the parameters of PID controller can be set as suggested in Table 4.2:

$$D_c(s) = k_P\left(1 + \frac{1}{T_I s} + T_D s\right)$$

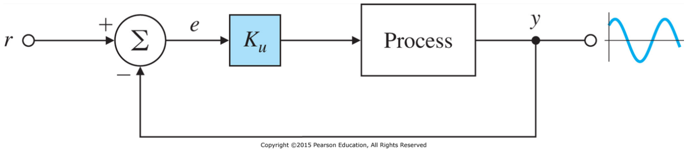
where

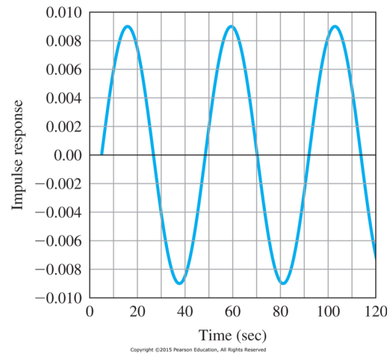
$$
\begin{aligned}
\text{P control}: k_P &= \frac{1}{RL} = \frac{\tau}{At_d} \\
\text{PI control}: k_P &= \frac{0.9}{RL} = 0.9\frac{\tau}{At_d} \qquad T_I = \frac{L}{0.3} = \frac{t_d}{0.3} \\
\text{PID control}: k_P &= \frac{1.2}{RL} = 1.2\frac{\tau}{At_d} \qquad T_I = 2L = 2t_d \qquad T_D = 0.5L = 0.5t_d
\end{aligned}
$$

Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

Type of Controller	Optimum Gain
P	$k_P = 1/RL$
PI	$\begin{cases} k_P = 0.9/RL \\ T_I = L/0.3 \end{cases}$
PID	$\begin{cases} k_P = 1.2/RL \\ T_I = 2L \\ T_D = 0.5L \end{cases}$

- Second method using proportional control (Ultimate Sensitivity Method)





  - Using the P control shown in Fig. 4.20, the P gain is increased until the system becomes marginally (or neutrally) stable and continuous oscillations just begin without variations of the amplitude. (Fig. 4.21)
  - The corresponding gain is defined as K_u (ultimate gain) and the period of oscillation is P_u (ultimate period).
  - The parameters of PID controller can be set as suggested in Table 4.3:

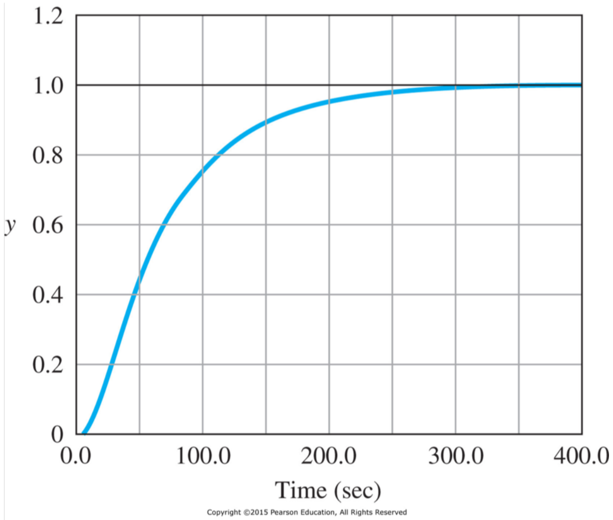
$$D_c(s) = k_P \left(1 + \frac{1}{T_I s} + T_D s\right)$$

where

$$\text{P control} : k_P = 0.5K_u$$

$$\text{PI control} : k_P = 0.45K_u \qquad T_I = \frac{P_u}{1.2}$$

$$\text{PID control} : k_P = 0.6K_u \qquad T_I = 0.5P_u \qquad T_D = 0.125P_u$$



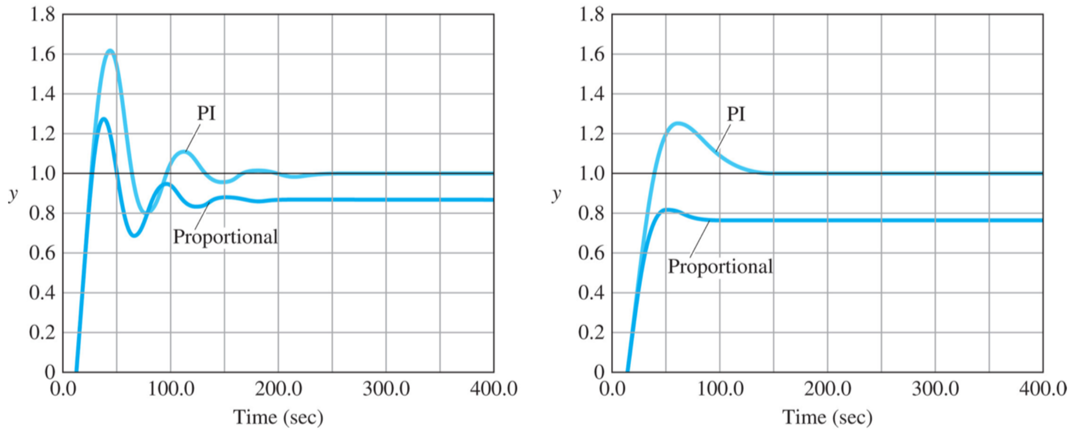
- (Example 4.9 Tuning of a Heat Exchanger : Quarter Decay Ratio, First Method) As the step response, let us assume that the process reaction curve was obtained as shown in Fig. 4.23 From the curve

$$R = \frac{A}{\tau} = \frac{1}{90}$$
$$L = t_d = 13$$

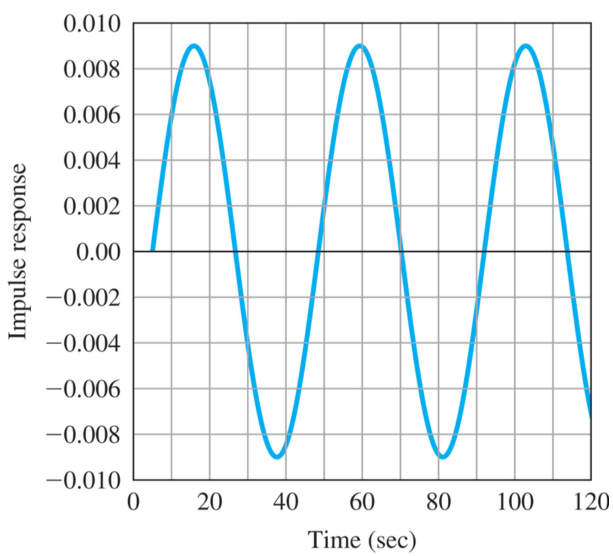
Thus the parameters of P and PI controllers are recommended as

$$\text{P control} : k_P = \frac{1}{RL} = \frac{90}{13} = 6.92$$
$$\text{PI control} : k_P = \frac{0.9}{RL} = 6.22 \qquad T_I = \frac{L}{0.3} = \frac{13}{0.3} = 43.3$$

Then we can get the response shown in Fig. 4.24(a), but it shows rather oscillatory behavior and considerable overshoot. If we reduce the gain k_P by a factor of 2, the overshoot and oscillatory behaviors are substantially reduced as shown in Fig. 4.24(b).



(a) (b)



- (Example 4.10 Tuning of a Heat Exchanger : Oscillatory Behavior, Second Method) Assume that we obtain the sustained oscillation after applying the P control as shown in Fig. 4.25.

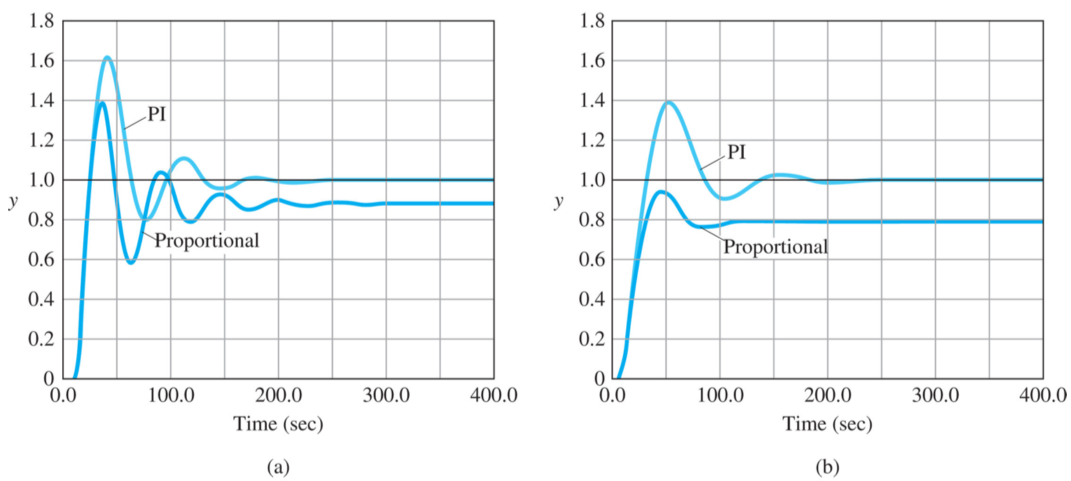
From the graph

$$K_u = 15.3 \qquad\qquad\qquad\qquad P_u = 42$$

Thus the parameters of P and PI controllers are recommended as

$$\textbf{P control} : k_P = 0.5K_u = 7.65$$

$$\textbf{PI control} : k_P = 0.45K_u = 6.885 \qquad T_I = \frac{P_u}{1.2} = 35$$



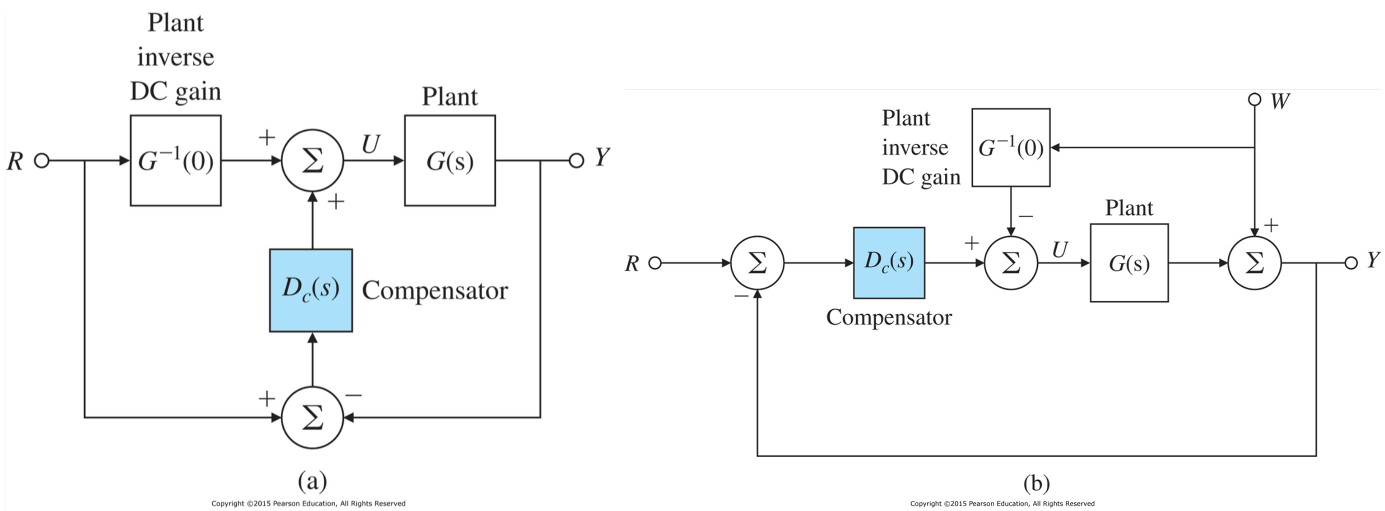
Then we obtained the step response as shown in Fig. 4.26(a). If we reduce k_P by 50%, the overshoot is substantially reduced as shown in Fig. 4.26(b).

7. Z-N tuning rules provide a good starting point but considerable fine tuning may still be needed.

4 Feedforward Control by Plant Model Inversion

- Integral control was introduced in order to reduce those errors to zero for steady disturbances or constant reference commands; however, integral control typically decreases the damping or stability of a system.

- One way to partly resolve this conflict is to provide some feedforward of the control that will eliminate the steady-state errors due to command inputs, because the stable feedforward term does not affect the entire stability.



- To eliminate the steady-state error, the inverse of the DC gain of the plant TF model, $G^{-1}(0)$ could be incorporated into the controller as shown in Fig. 4.27

- (Example 4.11 Feedforward Control for DC Motor) Consider the Fig. 4.27 with $G(s) = \frac{A}{s^2+a_1s+a_2}$.

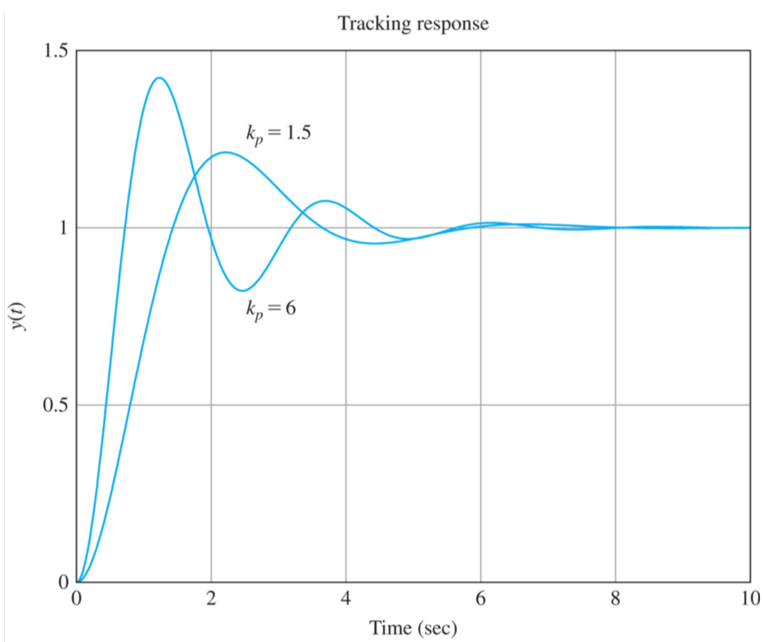
  1. Use feedforward control to eliminate the steady-state tracking error for step reference input when $k_P = 1.5, 6.0$

$$Y = G[G^{-1}(0)R + k_P(R - Y)] \quad \rightarrow \quad Y(s) = \frac{G(s)G^{-1}(0) + kG(s)}{1 + kG(s)} R(s)$$

$$y_{ss} = \frac{G(0)G^{-1}(0) + kG(0)}{1 + kG(0)} = 1$$

$$e_{ss} = 1 - y_{ss} = 0$$

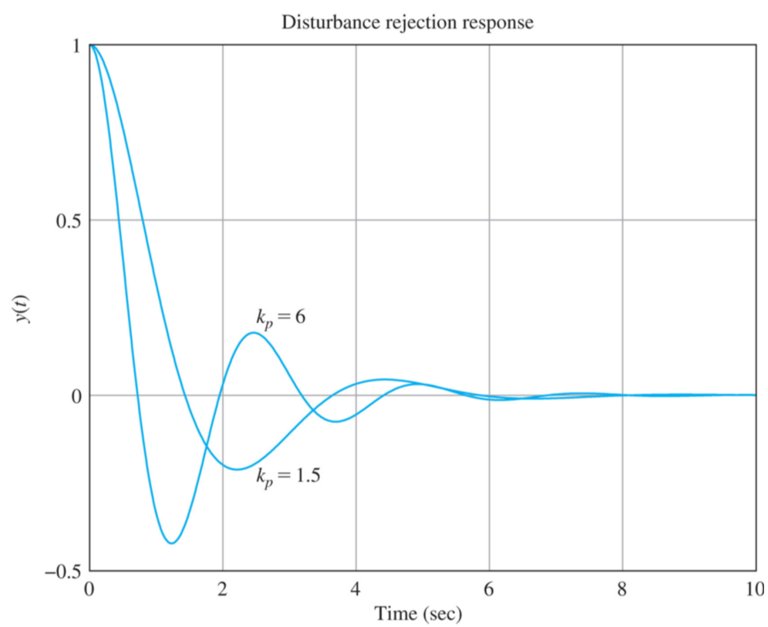
  As shown in Fig. 4.28, the steady-state errors become zero irrespective of the values of P gain.



2. Use feedforward control to eliminate the effect of a constant output disturbance signal on the output of the system when $k_P = 1.5, 6.0$ and $R(s) = 0$,

$$Y(s) = \frac{1 - G(s)G^{-1}(0)}{1 + kG(s)} W(s)$$
$$y_{ss} = \frac{1 - G(0)G^{-1}(0)}{1 + kG(0)} = 0$$

As shown in Fig. 4.29, the steady-state errors due to the step disturbance become zero irrespective of the values of P gain.



- This feedforward only controls the steady-state effect of the reference and disturbance inputs.

1. (Homework #4, Due : June 14, 2020) Solve and Submit 4.3, 4.4, 4.7, 4.10, 4.14, 4.16, 4.22, 4.27, 4.29, 4.32, 4.40, 4.44, 4.46, 4.48