제 11 장

Robot Control

- In tasks such as writing on a chalkboard, it must control forces in some directions (the force must press the chalk against the board) and motions in others (the motion must be in the plane of the board).
- The job of the robot controller is to convert the task specification to forces and torques at the actuators.
- Control strategies include (1) motion control, (2) force control, (3) hybrid motion-force control, (4) impedance control.
- Which of these behaviors is appropriate depends on both the task and the environment. For example, a force-control goal makes sense when the end-effector is in contact with something but not when it is moving in free space.
- The robot cannot independently control the motion and force in the same direction.
  - If the robot imposes a motion then the environment will determine the force
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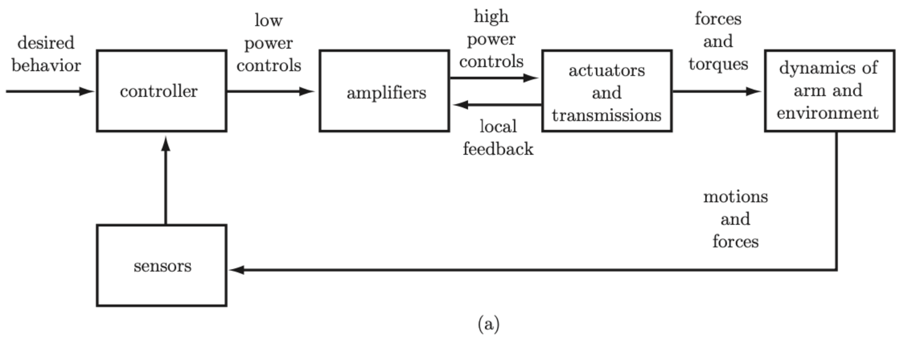
- Once we have chosen a control goal consistent with the task and environment, we can use feedback control to achieve it.

1 Control System Overview



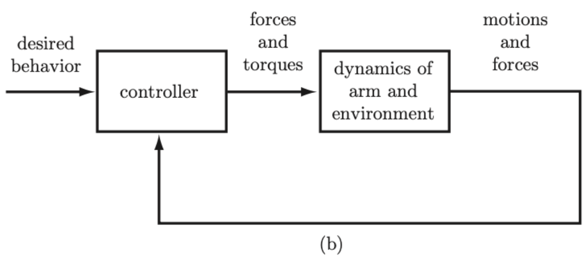


Figure 11.1: (a) A typical robot control system. An inner control loop is used to help the amplifier and actuator to achieve the desired force or torque. For example, a DC motor amplifier in torque control mode may sense the current actually flowing through the motor and implement a local controller to better match the desired current, since the current is proportional to the torque produced by the motor. Alternatively the motor controller may directly sense the torque by using a strain gauge on the motor's output gearing, and close a local torque-control loop using that feedback. (b) A simplified model with ideal sensors and a controller block that directly produces forces and torques. This assumes ideal behavior of the amplifier and actuator blocks in part (a). Not shown are the disturbance forces that can be injected before the dynamics block, or disturbance forces or motions injected after the dynamics block.

- Sensors are listed:
  - potentiometers, encoders, or resolvers for joint position and angle sensing
  - tachometers for joint velocity sensing
  - joint force-torque sensors
  - multi-axis force-torque sensors at the "wrist" between the end of the arm and the end-effector.
- The controller samples the sensors and updates its control signals to the actuators at a rate of hundreds to a few thousands of Hz.
- In our analysis we will ignore the fact that the sampling time is nonzero and treat controllers as if they were implemented in continuous time.
- Real robot systems are subject to
  - flexibility and vibrations in the joints and links
  - backlash at the gears and transmissions
  - actuator saturation limits
  - limited resolution of the sensors.

  These raise significant issues in design and control.

2 Error Dynamics

- Consider the controlled dynamics of a single joint and define the joint error to be

$$\theta_e(t) = \theta_d(t) - \theta(t)$$

- The differential equation governing the evolution of the joint error $\theta_e(t)$ of the controlled system is called the error dynamics.
- The purpose of the feedback controller is to create an error dynamics such that $\theta_e(t) \to 0$, or a small value, as $t \to \infty$.

2.1 Error Response

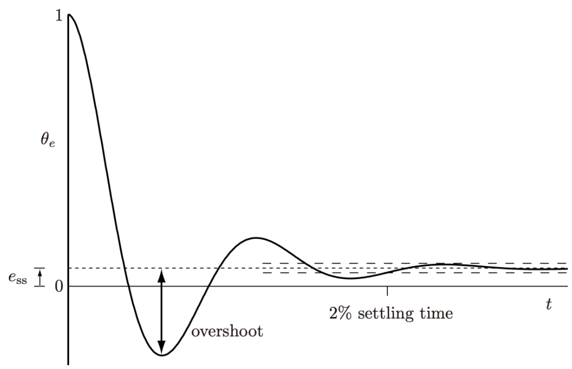


Figure 11.2: An example error response showing steady-state error e_{ss}, the overshoot, and the 2% settling time.

- A common way to test how well a controller works is to specify a nonzero initial error $\theta_e(0)$ and see how quickly, and how completely, the controller reduces the initial error.
- A typical error response $\theta_e(t)$ can be described by a transient response and a steady-state response.
  - The steady-state response is characterized by the steady-state error e_{ss}, which is the asymptotic error $\theta_e(t)$ as $t \to \infty$.
  - The transient response is characterized by the overshoot and (2%) settling time. The 2% settling time is the first time T such that $|\theta_e(t) - e_{ss}| <= 0.02(\theta_e(0) - e_{ss})$ for all $t \geq T$ (see the pair of long-dashed lines).
- A good error response is characterized by
  - little or no steady-state error
  - little or no overshoot
  - a short 2% settling time.

2.2 Linear Error Dynamics

- Consider the error dynamics described by linear ordinary differential equations of the form

$$a_p\theta_e^{(p)} + a_{p-1}\theta_e^{(p-1)} + \cdots + a_1\dot{\theta}_e + a_0\theta_e = c$$

  This is a pth-order differential equation

- The differential equation is homogeneous if the constant $c = 0$ and nonhomogeneous if $c \neq 0$.

- For homogeneous ($c = 0$) linear error dynamics, the pth-order differential equation can be rewritten as

$$\begin{aligned}\theta_e^{(p)} &= -\frac{a_{p-1}}{a_p}\theta_e^{(p-1)} - \cdots - \frac{a_1}{a_p}\dot{\theta}_e - \frac{a_0}{a_p}\theta_e \\ &= -a'_{p-1}\theta_e^{(p-1)} - \cdots - a'_1\dot{\theta}_e - a'_0\theta_e\end{aligned}$$

- This pth-order differential equation can be expressed as p coupled first-order differential equations by defining the vector $x = (x_1, \cdots, x_p)$, where

$$\begin{array}{lllll} x_1 = \theta_e & x_2 = \dot{\theta}_e & x_3 = \ddot{\theta}_e & \cdots & x_p = \theta_e^{(p-1)} \\ \dot{x}_1 = x_2 & \dot{x}_2 = x_3 & \dot{x}_3 = x_4 & \cdots & \dot{x}_p = -a'_0x_1 - a'_1x_2 - \cdots - a'_{p-1}x_p \end{array}$$

Then $\dot{x}(t) = Ax(t)$, where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_0' & -a_1' & -a_2' & \cdots & -a_{p-2}' & -a_{p-1}' \end{bmatrix} \in \Re^{p \times p}$$

- By analogy with the scalar first-order differential equation,

$$\dot{x} = ax(t) \quad \text{with a scalar } a \quad \rightarrow \quad x(t) = e^{at}x(0)$$
$$\dot{x} = Ax(t) \quad \text{with a matrix } A \quad \rightarrow \quad x(t) = e^{At}x(0)$$

where the scalar solution $x(t)$ goes to zero when $a < 0$ and the vector solution $x(t)$ tends to zero vector when $A < 0$ (all eigenvalues have negative real components)

- (Routh Stability Criterion) The eigenvalues of A are given by the roots of the characteristic polynomial of A, i.e.,

$$\det(sI - A) = s^p + a_{p-1}'s^{p-1} + a_2's^2 + a_1's + a_0' = 0$$

  - Necessary condition:

$$a_{p-1}' > 0 \quad \cdots \quad a_2' > 0 \quad a_1' > 0 \quad a_0' > 0$$

- Sufficient condition: the first column should be positive

$$\begin{array}{rllll} s^p : & 1 & a'_{p-2} & a'_{p-4} & \cdots \\ s^{p-1} : & a'_{p-1} & a'_{p-3} & a'_{p-5} & \cdots \\ s^{p-2} : & a'_{p-2} - \dfrac{a'_{p-3}}{a'_{p-1}} & a'_{p-4} - \dfrac{a'_{p-5}}{a'_{p-1}} & \cdots & \\ s^{p-3} : & \vdots & & & \end{array}$$

For example, $a'_{p-2} - \frac{a'_{p-3}}{a'_{p-1}} > 0$ is required.

- If each root of characteristic equation has a negative real component, we call the error dynamics stable.
- If any of the roots has a positive real component, the error dynamics are unstable, and the error $\|\theta_e(t)\|$ can grow without bound as $t \to \infty$.

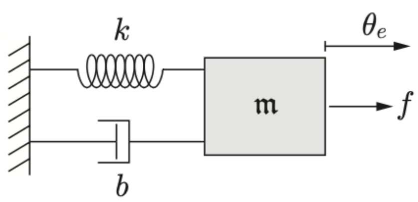


Figure 11.3: A linear mass–spring–damper.

- Consider a linear mass-spring-damper.
- The position of the mass m is θ_e and an external force f is applied to the mass.

$$\mathrm{m}\ddot{\theta}_e = f - b\dot{\theta}_e - k\theta_e \qquad \rightarrow \qquad \mathrm{m}\ddot{\theta}_e + b\dot{\theta}_e + k\theta_e = f$$

  This is second-order dynamics and it is stable by the Routh Stability Criterion, when $f = 0$.

- In the limit as the mass m approaches zero, the second-order dynamics reduces to the first-order dynamics

$$b\dot{\theta}_e + k\theta_e = f$$

  By the first-order dynamics, an external force generates a velocity rather than an acceleration.

First-Order Error Dynamics

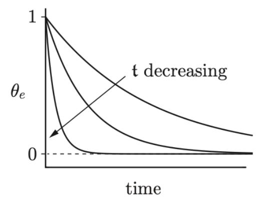


Figure 11.4: The first-order error response for three different time constants t.

- The first-order error dynamics with $f = 0$ can be written in the form using the Laplace Transform

$$\dot{\theta}_e(t) + \frac{k}{b}\theta_e = 0 \quad \rightarrow \quad \dot{\theta}_e(t) + \frac{1}{\mathrm{t}}\theta_e = 0 \quad \rightarrow \quad sE(s) - \theta_e(0) + \frac{1}{\mathrm{t}}E(s) = 0 \quad \rightarrow \quad E(s) = \frac{1}{s + 1/\mathrm{t}}\theta_e(0)$$

  where $\mathrm{t} = \frac{b}{k}$ is called the time constant of the first-order differential equation.

- The solution to the differential equation is

$$\theta_e(t) = e^{-\frac{t}{\mathrm{t}}}\theta_e(0)$$

- The time constant t is the time at which the first-order exponential decay has decayed to approximately 37% of its initial value.

- The steady-state error is zero, there is no overshoot in the decaying exponential error response, and the 2% settling time is determined by solving

$$\frac{\theta_e(t)}{\theta_e(0)} = 0.02 = e^{-\frac{t}{\mathrm{t}}} \quad \rightarrow \quad \ln 0.02 = -\frac{t}{\mathrm{t}} \quad \rightarrow \quad t = 3.91\mathrm{t} \quad \rightarrow \quad t \approx 4\mathrm{t}$$

- The response gets faster as the spring constant k increases or the damping constant b decreases.

Second-Order Error Dynamics

- The second-order error dynamics

$$\ddot{\theta}_e(t) + \frac{b}{\mathrm{m}}\dot{\theta}_e(t) + \frac{k}{\mathrm{m}}\theta_e(t) = 0$$

can be written in the standard second-order form

$$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$$

where $\omega_n = \sqrt{\frac{k}{\mathrm{m}}}$ is called the natural frequency and $\zeta = \frac{b}{2\sqrt{k\mathrm{m}}}$ is called the damping ratio.

- Let us define $x_1 = \theta_e$ and $x_2 = \dot{\theta}_e$, then we have

$$\dot{x} = Ax \qquad \text{with} \qquad A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}$$

The characteristic equation becomes

$$\det(sI - A) = 0 \qquad \rightarrow \qquad s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

and its roots are

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} \qquad\qquad s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

- There are three types of solutions $\theta_e(t)$ to the differential equation, depending on whether the roots s_1, s_2 are real and unequal ($\zeta > 1$), real and equal ($\zeta = 1$), or complex conjugates ($\zeta < 1$).

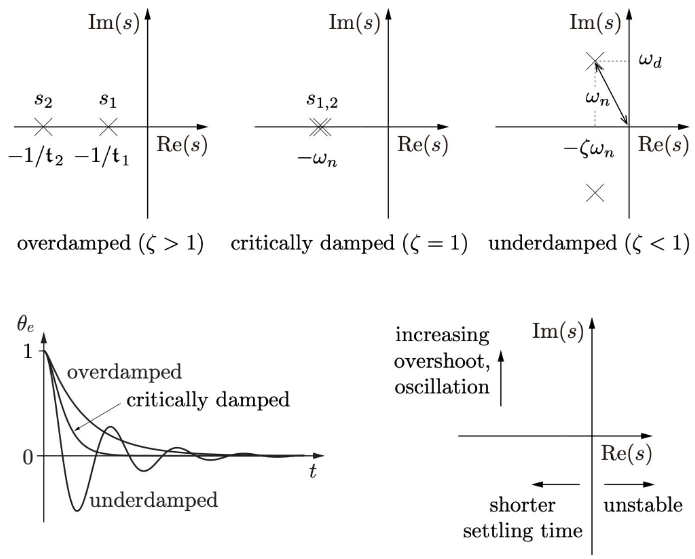


Figure 11.5: (Top) Example root locations for overdamped, critically damped, and underdamped second-order systems. (Bottom left) Error responses for overdamped, critically damped, and underdamped second-order systems. (Bottom right) Relationship of the root locations to properties of the transient response.

1. Overdamped: $\zeta > 1$. The roots s1,2 are real and distinct, and the solution is

$$\theta_e(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

  - The response is the sum of two decaying exponentials, with time constants $t_1 = -\frac{1}{s_1}$ (slow) and $t_2 = -\frac{1}{s_2}$.

  - The initial conditions for the (unit) error response are $\theta_e(0) = 1$ and. $\dot{\theta}_e(0) = 0$, and the constants c_1 and c_2 can be calculated as

$$c_1 = \frac{1}{2} + \frac{\zeta}{2\sqrt{\zeta^2 - 1}} \qquad c_2 = \frac{1}{2} - \frac{\zeta}{2\sqrt{\zeta^2 - 1}}$$

2. Critically damped: $\zeta = 1$. The roots $s_1, s_2 = -\omega_n$ are equal and real, and the solution is

$$\theta_e(t) = (c_1 + c_2 t)e^{-\omega_n t}$$

   - The time constant of the decaying exponential is $\mathrm{t} = \frac{1}{\omega_n}$.
   - For the error response with $\theta_e(0) = 1$ and $\dot{\theta}_e(0) = 0$,

$$c_1 = 1 \qquad\qquad c_2 = \omega_n$$

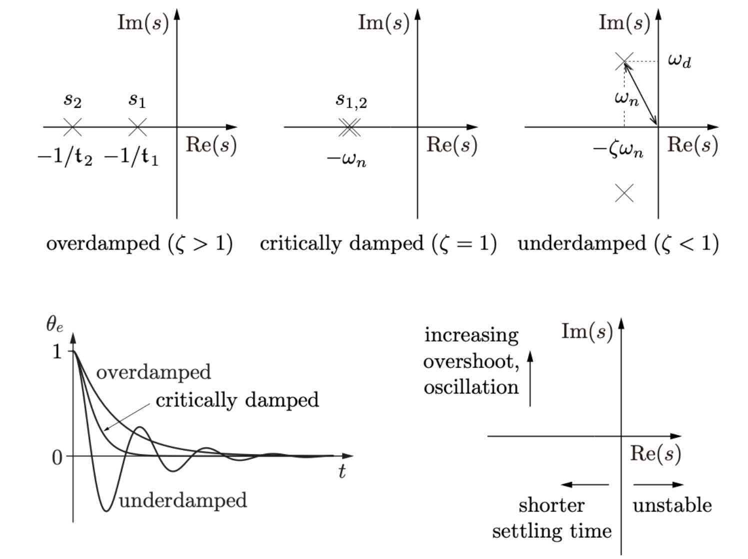
3. Underdamped: $\zeta < 1$. The roots are complex conjugates at $s_{1,2} = -\zeta\omega_n \pm j\omega_d$ where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency. The solution is

$$\theta_e(t) = e^{-\zeta\omega_n t}(c_1 \cos\omega_d t + c_2 \sin\omega_d t)$$

   - A decaying exponential (time constant $\mathrm{t} = \frac{1}{\zeta\omega_n}$ multiplied by a sinusoid.
   - For the error response with $\theta_e(0) = 1$ and $\dot{\theta}_e(0) = 0$,

$$c_1 = 1 \qquad\qquad c_2 = \frac{\zeta}{\sqrt{1-\zeta^2}}$$

- Example root locations for the overdamped, critically damped, and underdamped cases, as well as their error responses $\theta_e(t)$, are shown in the following figure.



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- The figure also shows the relationship between the root locations and properties of the transient response:
  - roots further to the left in the complex plane correspond to shorter settling times,
  - roots further away from the real axis correspond to greater overshoot and oscillation.
- In the case of underdamped, there is the peak of the overshoot, and it occurs at the time t_p, and substituting this into the underdamped error response, we get

$$t_p = \frac{\pi}{\omega_d} \qquad \rightarrow \qquad \theta_e(t_p) = -e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

- Thus $\zeta = 0.1$ gives an overshoot of 73%, $\zeta = 0.5$ gives an overshoot of 16%, and $\zeta = 0.8$ gives an overshoot of 1.5%.