- (Example 6.2) Bode plot of the lead compensation, which is equivalent to

$$D_c(s) = K\frac{Ts+1}{\alpha Ts+1} \qquad \text{for} \quad \alpha < 1$$

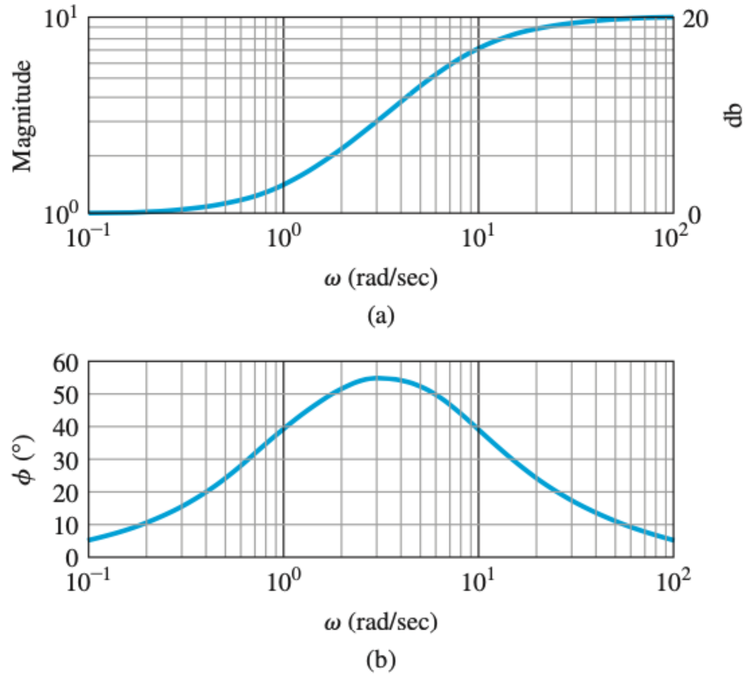
$$\begin{aligned}
|D_c(j\omega)|_{dB} &= 20\log_{10}|D_c(j\omega)| \\
&= 20\log_{10}\left|K\frac{Ts+1}{\alpha Ts+1}\right| \\
&= 20\log_{10}|K| + 20\log_{10}|1+j\omega T| - 20\log_{10}|1+j\omega\alpha T| \\
&= 20\log_{10}|K| + 20\log_{10}\sqrt{1+\omega^2T^2} - 20\log_{10}\sqrt{1+\omega^2\alpha^2T^2}
\end{aligned}$$

$$\begin{aligned}
\angle D_c(j\omega) &= \angle K + \angle(1+j\omega T) - \angle(1+j\omega\alpha T) \\
&= 0 + \tan^{-1}(\omega T) - \tan^{-1}(\omega\alpha T)
\end{aligned}$$

Use $K = 1$, $T = 1$ and $\alpha = 0.1$

$$\begin{aligned}
&\omega = 0.1 \quad \rightarrow \quad |D_c(j\omega)|_{dB} \approx 0 \quad \text{and} \quad \angle D_c(j\omega) \approx 0 \\
&\omega = \frac{1}{T} = 1 \quad \rightarrow \quad |D_c(j\omega)|_{dB} \approx 3 \quad \text{and} \quad \angle D_c(j\omega) \approx 45^\circ \\
&\omega = \frac{1}{\alpha T} = 10 \quad \rightarrow \quad |D_c(j\omega)|_{dB} \approx 17 \quad \text{and} \quad \angle D_c(j\omega) \approx 45^\circ \\
&\omega = 100 \quad \rightarrow \quad |D_c(j\omega)|_{dB} \approx 20 \quad \text{and} \quad \angle D_c(j\omega) \approx 0^\circ
\end{aligned}$$

Figure 6.2
(a) Magnitude;
(b) phase for the lead compensation in Example 6.2



- (Example 6.3) Bode plot of the following transfer function

$$G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)} \qquad \rightarrow \qquad G(j\omega) = \frac{2(1+2j\omega)}{(j\omega)(1+0.1j\omega)(1+0.02j\omega)}$$

1. 크기:

$$|G(j\omega)|_{dB} = 20\log_{10}|2| + 20\log_{10}\sqrt{1+(2\omega)^2} - 20\log_{10}|\omega| - 20\log_{10}\sqrt{1+(0.1\omega)^2} - 20\log_{10}\sqrt{1+(0.02\omega)^2}$$

2. 위상:

$$\angle G(j\omega) = 0 + \tan^{-1}(2\omega) - 90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}(0.02\omega)$$

3. see Fig. 6.9

- (Example 6.4)
- (Example 6.5)
- (Example 6.6)

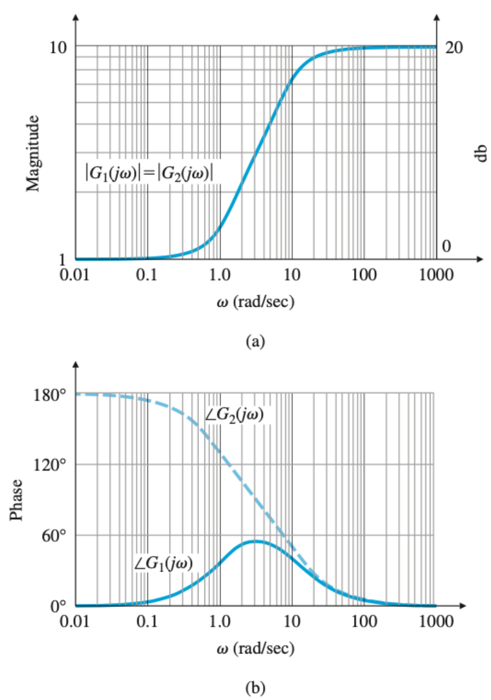
- (Nonminimum-Phase Systems) 최소위상함수와 비최소위상함수

  1. s 평면의 오른쪽 반평면에 극이나 영점을 갖지 않는 전달함수는 최소위상 시스템이며, s 평면의 오른쪽 반평면에 극이나 영점을 갖는 전달함수는 비최소위상 전달함수이다.
  2. 같은 크기 특성을 가진 시스템에 대해서, 최소위상 전달함수의 위상각의 범위가 항상 최소이며, 동일 크기 특성을 갖는 비최소위상 전달함수의 위상각의 범위는 이 최소값보다 크다.
  3. 최소위상 시스템에 대해서는 크기 곡선만으로 전달함수를 유일하게 결정할 수 있다. 비최소위상 시스템은 불가능하다.

$$G_1(s) = 10\frac{s+1}{s+10} \quad \text{and} \quad G_2(s) = 10\frac{s-1}{s+10}$$

Both TFs have the same magnitude for all frequencies, but the phases of the two TFs are drastically different

Figure 6.12
Bode plot of minimum- and nonminimum-phase systems: for (a) magnitude; (b) phase



4. 다음의 전달함수를 분석해 보자

$$G(s) = \frac{1}{1 - sT} \qquad \rightarrow \qquad G(j\omega) = \frac{1}{1 - j\omega T}$$

- **for positive** T

$$\angle G(j\omega) = \tan^{-1}\omega T \qquad \rightarrow \qquad \angle G(j\omega) : 0 \rightarrow 90^\circ$$

$\rightarrow$ 위상이 **0**도에서 **90**도 상승하는 경우는 $1 + j\omega T$도 있어서, 어느것인지 구분되지 않는다.

- **for negative** T

$$\angle G(j\omega) = -\tan^{-1}(-\omega T) \qquad \rightarrow \qquad \angle G(j\omega) : 0 \rightarrow -90^\circ$$

5. 비최소위상 전달함수의 보데 선도로는 안정도 해석을 할 수 없다. (그러나 **Nyquist** 선도는 안정도 판별이 가능하다)

1.2 Steady-State Errors

- Consider the open-loop transfer function:

$$KG(s) = K\frac{(1+T_1 s)}{s^n(1+T_a s)(1+2\zeta s/\omega_n + s^2/\omega_n^2)} \quad \rightarrow \quad \text{at low frequency} \quad KG(j\omega) \approx \frac{K_o}{(j\omega)^n}$$

- The larger the value of the magnitude of the low-frequency asymptote, the lower the steady-state errors will be for the closed-loop system.

- For unity-feedback system with $n = 0$ (Type 0 system), the low-frequency asymptote is a constant, and the gain K_o of the open-loop system is equal to the position-error constant K_p.

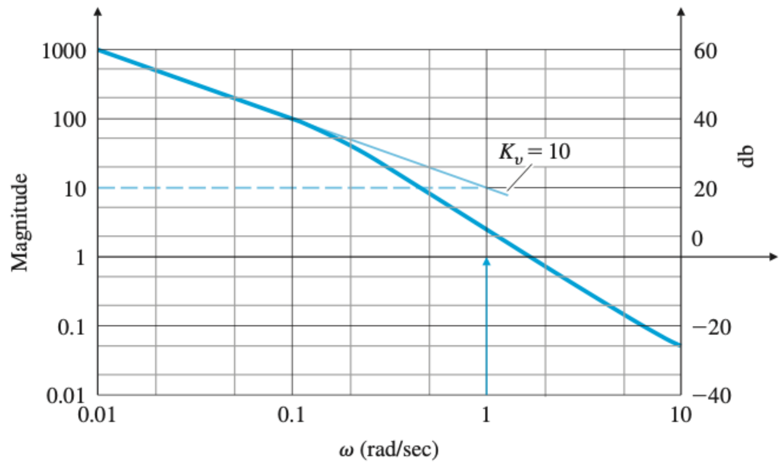
$$K_p = K_o \qquad \rightarrow \qquad e_{ss} = \frac{1}{1+K_p} \quad \text{with a unit-step input}$$

- For unity-feedback system with $n = 1$ (Type 1 system), the low frequency asymptote has a slope of -1 with K_o/ω, directly from the Bode magnitude plot. Then the velocity-error constant

$$K_v = K_o \qquad \rightarrow \qquad e_{ss} = \frac{1}{K_v} \quad \text{with a unit-ramp input}$$

- The easiest way of determining the value of K_v in a Type 1 system is to read the magnitude of the low-frequency asymptote at $\omega = 1$, because this asymptote is $A(\omega) = K_v/\omega$

Figure 6.13
Determination of K_v from the Bode plot for the system $KG(s) = \frac{10}{s(s+1)}$



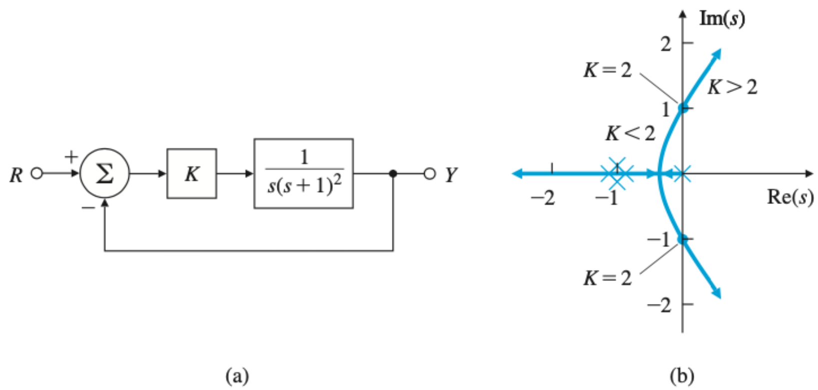
- (Example 6.7) Find K_v of the unity-feedback system having the system $KG(s) = \frac{10}{s(s+1)}$?

2 Neutral Stability

- If we know closed-loop TF, then we can check the stability easily by inspecting the positions of poles.
- If we know open-loop TF, then we can check the stability by using Root Locus. All points on the locus have the property that

$$|KG(s)| = 1 \qquad \text{and} \qquad \angle G(s) = \pm 180^\circ$$

Figure 6.14
Stability example:
(a) system definition;
(b) root locus



- At the point of neutral stability we see that these RL conditions hold for $s = j\omega$, so

$$|KG(j\omega)| = 1 \qquad \text{and} \qquad \angle G(j\omega) = \pm 180^\circ$$

- For stability, the following two conditions should be satisfied

$$\text{when} \qquad \angle G(j\omega) = -180^\circ \qquad \rightarrow \qquad |KG(j\omega)| < 1$$

Figure 6.15
Frequency-response magnitude and phase for the system in Fig. 6.14