10 Integral Control and Robust Tracking

10.1 Integral Control

- The choice of $\bar{N}$ will result in zero steady-state error to a step command, but the result is not robust because any change in plant parametrs will cause the error to be nonzero.
- It shows how the integral control can be introduced by a direct method of adding the integral of the system error to the dynamic equations.
- For the system

$$\dot{x} = Ax + Bu + B_1 w \qquad\qquad y = Cx$$

we can feed back the integral of the error, $e = y - r$, as well as the state of the plant, x, by augmenting the plant state with the extra state x_I, which obeys the differential equation

$$\dot{x}_I = Cx - r = e$$

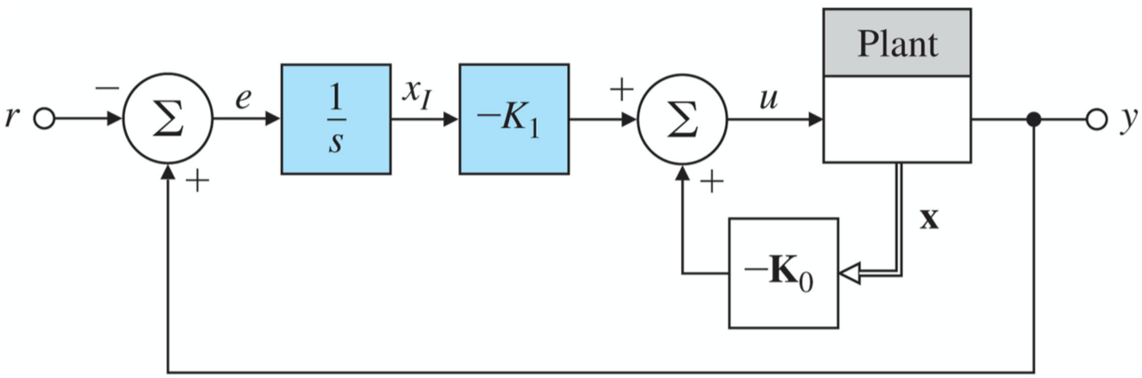
Thus

$$x_I = \int e(\tau)d\tau$$

- The augmented state equations and the feedback law become

$$\begin{bmatrix} \dot{x}_I \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \begin{bmatrix} x_I \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} w$$

$$u = -\begin{bmatrix} K_1 & K_0 \end{bmatrix} \begin{bmatrix} x_I \\ x \end{bmatrix}$$



- (Example 7.34) Consider the motor speed system described by

$$\frac{Y(s)}{U(s)} = \frac{1}{s+3}$$

that is, $A = -3$, $B = 1$ and $C = 1$. Design the system to have integral control and two poles at $s = -5$. Design an estimator with pole at $s = -10$. The disturbance enters at the same place as the control.

(Solution) The augmented state equation and the feedback law become

$$\begin{bmatrix} \dot{x}_I \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_I \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

$$u = -\begin{bmatrix} K_1 & K_0 \end{bmatrix} \begin{bmatrix} x_I \\ \hat{x} \end{bmatrix}$$

Therefore, we can find K from

$$\alpha_c(s) = \det\left(sI - \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} K_1 & K_0 \end{bmatrix} \right) = (s+5)^2$$

$$s^2 + (3 + K_0)s + K_1 = s^2 + 10s + 25$$

Consequently

$$K = \begin{bmatrix} K_1 & K_0 \end{bmatrix} = \begin{bmatrix} 25 & 7 \end{bmatrix} \qquad \rightarrow \qquad u = -25x_I - 7\hat{x} = -25 \int (y - r)\ dt - 7\hat{x}$$

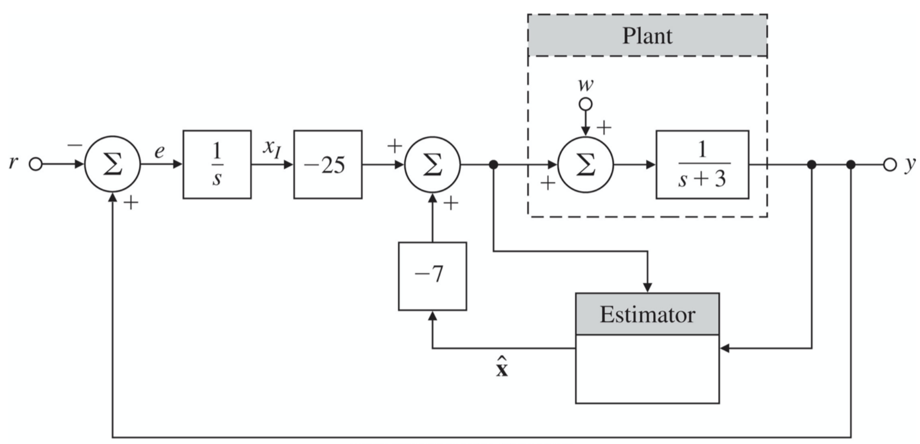
The estimator gain is obtained by

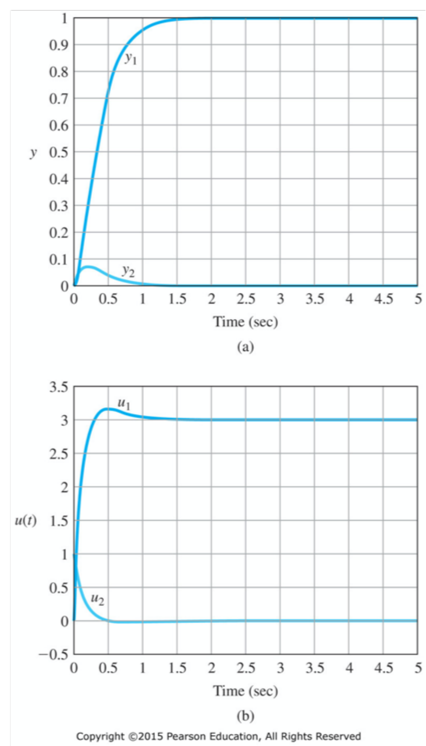
$$\alpha_e(s) = \det(s - A + LC) = s + 3 + L = s + 10 \quad \rightarrow \quad L = 7$$

The estimator equation is of the form:

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + Bu + L(y - C\hat{x}) \\ &= -10\hat{x} + u + 7y\end{aligned}$$

See the Fig. 7.54 for the integral control structure and its results (step-input response y_1 and step-disturbance response y_2) are shown in Fig. 7.55





10.2 Robust Tracking Control: The Error-Space Approach

1. A more analytical approach is presented to giving the control system the ability to track a nondecaying input and to reject a nondecaying disturbance such as step, ramp, or sinusoidal input.

2. The error approaches zero even if the output is following a nondecaying, or even a growing, command (such as a ramp signal) and even if some parameters change (robustness property).

3. Suppose we have the system state equations:

$$\dot{x} = Ax + Bu + B_1 w \qquad\qquad y = Cx$$

   and a reference signal that is known to satisfy a specific differential equation such as

$$\ddot{r} + \alpha_1 \dot{r} + \alpha_2 r = 0$$

   in addition, we assume the disturbance to satisfy exactly the same equation:

$$\ddot{w} + \alpha_1 \dot{w} + \alpha_2 w = 0$$

4. The tracking error is defined as

$$e = y - r \qquad \rightarrow \qquad r = y - e$$

   and we can know the following equality from the reference signal dynamics

$$\ddot{r} + \alpha_1 \dot{r} + \alpha_2 r = 0 \qquad \rightarrow \qquad \begin{aligned} \ddot{e} + \alpha_1 \dot{e} + \alpha_2 e &= \ddot{y} + \alpha_1 \dot{y} + \alpha_2 y \\ &= C(\ddot{x} + \alpha_1 \dot{x} + \alpha_2 x) \end{aligned}$$

5. Now replace the plant state vector with the error-space state and the control with the error-space control, respectively,

$$\xi = \ddot{x} + \alpha_1 \dot{x} + \alpha_2 x \qquad \mu = \ddot{u} + \alpha_1 \dot{u} + \alpha_2 u$$

6. With these definitions, we can replace the error dynamics

$$\ddot{e} + \alpha_1 \dot{e} + \alpha_2 e = C\xi$$

and the state equation for ξ is given by

$$\begin{aligned}\dot{\xi} &= \dddot{x} + \alpha_1 \ddot{x} + \alpha_2 \dot{x} = (A\ddot{x} + B\ddot{u} + B_1\ddot{w}) + \alpha_1(A\dot{x} + B\dot{u} + B_1\dot{w}) + \alpha_2(Ax + Bu + B_1 w) \\ &= A(\ddot{x} + \alpha_1 \dot{x} + \alpha_2 x) + B(\ddot{u} + \alpha_1 \dot{u} + \alpha_2 u) + B_1(\ddot{w} + \alpha_1 \dot{w} + \alpha_2 w) \\ &= A\xi + B\mu\end{aligned}$$

7. By combining the errors $e, \dot{e}$ and the error-space state ξ, we have

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \\ \xi \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha_2 & -\alpha_1 & C \\ 0 & 0 & A \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \\ \xi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}\mu$$

and, furthermore, we can design the control rule of the form

$$\mu = -\begin{bmatrix} K_2 & K_1 & K_0 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \\ \xi \end{bmatrix}$$

8. Let us find out the actual control input from μ

$$\begin{aligned} \ddot{u} + \alpha_1 \dot{u} + \alpha_2 u &= -K_2 e - K_1 \dot{e} - K_0 \xi \\ &= -K_2 e - K_1 \dot{e} - K_0 (\ddot{x} + \alpha_1 \dot{x} + \alpha_2 x) \end{aligned}$$

In other words, we have

$$(\ddot{u} + K_0 \ddot{x}) + \alpha_1 (\dot{u} + K_0 \dot{x}) + \alpha_2 (u + K_0 x) = -K_2 e - K_1 \dot{e}$$

and, if we assume $\dot{r} = 0$ such as step input, then there is no twice differentiations such as $\ddot{x}, \ddot{u}, \ddot{r}$, and $\alpha_1 = 1$ and $\alpha_2 = 0$. Thus we have

$$\dot{u} + K_0 \dot{x} = -K_2 e - K_1 \dot{e} \qquad \rightarrow \qquad \therefore \quad u = -K_2 \int (y - r) dt - K_1 (y - r) - K_0 x$$

9. (Example 7.35)

10. (Example 7.36) For the system

$$H(s) = \frac{1}{s+3}$$

with the state-variable description

$$A = -3 \qquad B = 1 \qquad C = 1$$

construct a controller with poles at $s = -5$ to track an input that satisfies $\dot{r} = 0$.
(solution) The error-space system is

$$\begin{bmatrix} \dot{e} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} e \\ \xi \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mu$$

where $e = y - r$, $\xi = \dot{x} = \dot{y}$ and $\mu = \dot{u}$.
If we take the desired characteristic equation to be

$$\alpha_c(s) = (s+5)^2 = s^2 + 10s + 25 = s^2 + (3 + K_0)s + K_1$$

where $\mu = -K_1 e - K_0 \xi$, where $K_1 = 25$ and $K_0 = 7$.
Thus we have the control input

$$\mu = \dot{u} \qquad \rightarrow \qquad u = -25 \int (y - r)dt - 7y$$

In addition,

$$u = -25\int(y-r)dt - 7y + Nr$$

if we choose $N = 8$, then

$$u = -25\int(y-r)dt - 7(y-r) + r$$

(7장 숙제) 수업시간에 학습한 내용에 상응하는 61개의 문제 중 10개 풀어 기말고사에 제출