Solutions of Midterm Exam

Subject : Control System Engineering 1, Lecturer : Prof. Youngjin Choi,
Date : May 26, 2020 (Contact e-mail : cyj@hanyang.ac.kr)

Problem 1 (20pt) Solve the following differential equation by means of the Laplace Transform:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = -e^{-3t} \cdot 1(t)$$

with $\frac{dy}{dt}(0) = 1$ and $y(0) = 0$, where $1(t)$ is a unit-step function.

Solution of Problem 1 (20pt)

1. Take the LT

$$\begin{aligned}
[s^2Y(s) - sy(0) - y'(0)] + 3[sY(s) - y(0)] + 2Y(s) &= -\frac{1}{s+3} \\
[s^2Y(s) - 1] + 3sY(s) + 2Y(s) &= -\frac{1}{s+3} \\
(s^2 + 3s + 2)Y(s) &= -\frac{1}{s+3} + 1 \\
(s+1)(s+2)Y(s) &= \frac{s+2}{s+3}
\end{aligned}$$

   Thus we have

$$Y(s) = \frac{1}{(s+1)(s+3)}$$

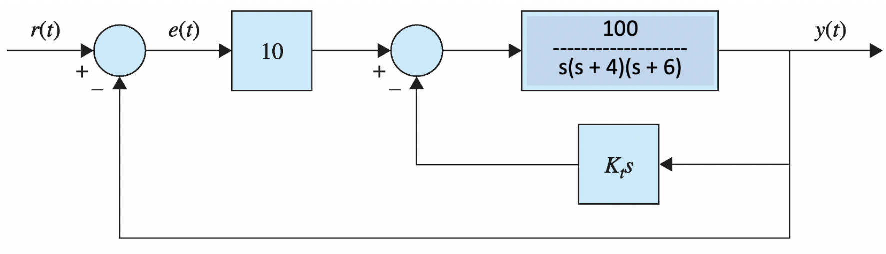
2. Use partial fraction method

$$Y(s) = \frac{0.5}{s+1} + \frac{-0.5}{s+3}$$

   Take inverse LT, then we have

$$\therefore \quad y(t) = 0.5e^{-t} - 0.5e^{-3t} \quad \text{for } \; t \geq 0$$

Problem 2 (20pt) Find the range of the tachometer constant K_t so that the system is stable ?



Solution of Problem 2 (20pt)

1. The characteristic equation is

$$
\begin{aligned}
1 + 10 \times \frac{100}{s(s+4)(s+6) + 100K_t s} &= 0 \\
s(s+4)(s+6) + 100K_t s + 1000 &= 0 \\
s^3 + 10s^2 + (24 + 100K_t)s + 1000 &= 0
\end{aligned}
$$

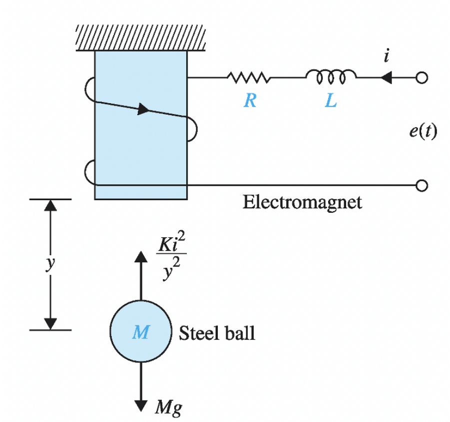
2. Routh-Hurwitz method is applied

$$
\begin{array}{ll}
s^3 : 1 & 24 + 100K_t \\
s^2 : 10 & 1000 \\
s^1 : 24 + 100K_t - \dfrac{1000}{10} & \\
s^0 : 1000 &
\end{array}
$$

Thus we have

$$100K_t - 76 > 0 \quad \rightarrow \quad \therefore \quad K_t > 0.76$$

Problem 3 (20pt) 다음 그림과 같은 구-현가장치 제어시스템을 고려해 보자. 강철구는 전자석에 의하여 생긴 전자기력 $\frac{Ki(t)^2}{y(t)^2}$에 의하여 중력 Mg에 대항하여 공중에 붙들려 있다. 입력전압 $e(t)$에 의하여 자석에 흐르는 전류를 제어하는 것을 목적으로 한다. 코일의 저항은 R, 그리고 인덕턴스는 $L(y) = \frac{L}{y(t)}$이며, 여기서 K, M, R과 L은 상수이다. 전기 및 역학 비선형 미분 방정식을 유도하라?



Solution of Problem 3 (20pt)

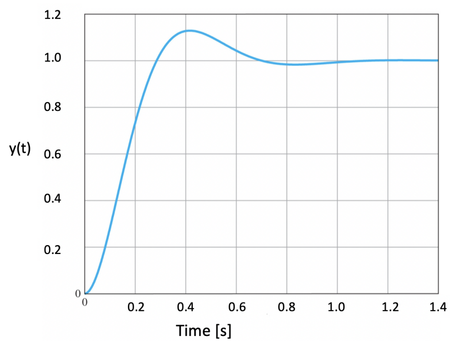
구의 관성력은 전류의 제곱에 비례하는 전자석의 힘과 중력사이의 평형 힘과 같다

$$M\ddot{y} = Mg - K\frac{i^2}{y^2} \qquad e = Ri + \frac{d}{dt}\left(\frac{Li}{y}\right) = Ri - \frac{Li}{y^2}\dot{y} + \frac{L}{y}\frac{di}{dt}$$

Problem 4 (20pt) For the given control system,

$$\frac{Y(s)}{R(s)} = \frac{1}{Ls^2 + Rs + \frac{1}{C}}$$

we have obtained the unit-step response $R(s) = \frac{1}{s}$ at the output terminal $y(t)$. What are the values of L, R and C?



Solution of Problem 4 (20pt)

1. By applying final value theorem, we have

$$y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{1}{Ls^2 + Rs + \frac{1}{C}} = C = 1$$

2. The maximum overshoot is about 13%

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.13 \quad \rightarrow \quad \zeta = \frac{|\ln M_p|}{\sqrt{\pi^2 + (\ln M_p)^2}} = \frac{2.04}{\sqrt{\pi^2 + (2.04)^2}} = 0.544$$

3. The settling time is about 0.9[s]

$$t_s = \frac{4.6}{\zeta\omega_n} = 0.9 \quad \rightarrow \quad \omega_n = 9.4$$

Therefore,

$$\frac{1}{LC} = \omega_n^2 = 88.36 \quad \rightarrow \quad L = 0.0113$$
$$\frac{R}{L} = 2\zeta\omega_n = 10.23 \quad \rightarrow \quad R = 0.1156$$

Problem 5 (20pt) 다음과 같은 역진자 모델을 고려해 보자. 수치 ($I = 1$, $mgl = 1$, and $T_c = 0$) 데이터를 적용하면 다음과 같은 아주 간단한 비선형 미분방정식을 얻을 수 있다.

$$I\ddot{\theta} + mgl\sin\theta = T_c \qquad \rightarrow \qquad \ddot{\theta} + \sin\theta = 0$$

초기 진자의 각도가 $\theta(0) = 60°$이고 초기 진자의 각속도가 $\dot{\theta}(0) = 0$ 였을 때, 임의의 시간 t에서의 각속도 $\dot{\theta}(t)$를 각도 $\theta(t)$의 함수로 나타내라?

Solution of Problem 5 (20pt)

1. 연쇄법칙을 적용해 보자

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = -\sin\theta$$

그러므로

$$\begin{aligned} \dot{\theta}d\dot{\theta} &= -\sin\theta d\theta \\ \int_{\dot{\theta}(0)}^{\dot{\theta}(t)} \dot{\theta}d\dot{\theta} &= -\int_{\theta(0)}^{\theta(t)} \sin\theta d\theta \\ \frac{1}{2}[\dot{\theta}(t)^2 - 0] &= \cos\theta(t) - \cos 60° = \cos\theta(t) - 0.5 \\ \therefore \quad \dot{\theta}(t) &= \sqrt{2\cos\theta(t) - 1} \end{aligned}$$