제 7 장

State-Space Design

- 지금까지 배운 제어기 설계 방법: Root Locus, Frequency Response
- 설계사양 - 상대안정도, 정상상태오차, 과도응답특성 (최대오버슈트, 상승시간, 지연시간, 정착시간), 주파수응답특성 (공진주파수, 공진봉우리, 대역폭, 이득여유, 위상여유), 파라미터 민감도 (강인성), 외란제거 등
  - 과도응답특성 (최대오버슈트, 상승시간, 지연시간, 정착시간) - 임펄스응답, 계단응답, 포물선응답 등
  - 주파수응답특성 (공진주파수, 공진봉우리, 대역폭, 이득여유, 위상여유) - Nyquist, Bode, Nichols
- Third major method of designing feedback control systems: the STATE-SPACE method.

1 Advantages of State-Space

- State-space control design is the technique in which the control engineer designs a dynamic compensation by working directly with the state-variable description of the system.

- Ordinary differential equations (ODEs) of physical dynamic systems can be manipulated into state-variable form.

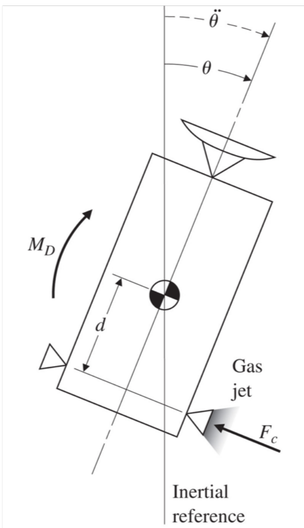
- 현대제어이론 vs 고전제어이론
  - 고전 제어이론 (transform method) - 선형, 시불변, 단일입력-단일출력 시스템에만 적용. 주로 주파수영역 접근방법 혹은 근궤적법으로 제어기 설계
  - 현대 제어이론 (state-space method) - 선형 or 비선형, 시불변 or 시변, 다수입력-다수출력 시스템에도 적용. 주로 상태공간 접근법으로 제어기 설계

- 정의
  - 상태 (state) - 동적 시스템의 상태는 $t = t_0$에서 변수를 알고, $t \geq t_0$에서 입력을 알면, $t \geq t_0$에서 시스템의 거동을 완전히 결정할 수 있을 때, 이러한 변수들의 최소 집합
  - 상태변수 (state variable) - 동적 시스템의 상태를 결정할 수 있는 최소 개수의 변수들. 상태변수는 물리적으로 측정될 수 있거나 관측될 수 있는 양일 필요는 없다.
    state variables: position (potential energy), velocity (kinetic energy), capacity voltage (electric energy), inductor current (magnetic energy), thermal temperature (thermal energy)
  - 상태벡터 (state vector) - 주어진 시스템의 거동을 표현하기 위해 n개 상태변수가 필요하다면, n개 성분을 벡터로 생각할 수 있는데, 이러한 벡터를 상태벡터라고 한다.

- 상태공간 방정식 (state-space equation) - 동적 시스템을 모델링할 때, 변수들은 입력변수, 출력변수, 상태변수의 3종류로 구성된다. 시스템의 상태공간 표현은 여러가지 있을 수 있으나, 상태변수의 개수만은 일정하다. 시스템의 동특성을 완전히 정의할 수 있는 상태변수의 수는 시스템에 포함된 적분기의 수와 동일하다.

2 System Descriptions in State-Space

- The motion of any finite dynamic system can be expressed as a set of first-order ODEs.
- (Example 7.1) Consider Fig. 2.8 in the textbook with $M_D = 0$, we have the attitude problem of the satellite.



$$I\ddot{\theta} = F_c d \qquad u = F_c \qquad y = \theta$$

Let us assign states as follows:

$$x_1 = \theta \qquad x_2 = \dot{\theta}$$

Take the time derivatives

$$\dot{x}_1 = \dot{\theta} = x_2 \qquad \dot{x}_2 = \ddot{\theta} = \frac{d}{I}u$$

Let us obtain state-variable equation as the vector equation:

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{d}{I} \end{bmatrix} u$$

$$\dot{x} = Ax + Bu$$

and the output equation is

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u$$

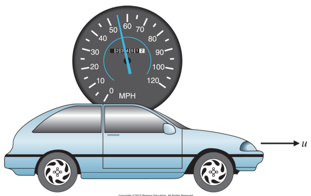
$$y = Cx + Du$$

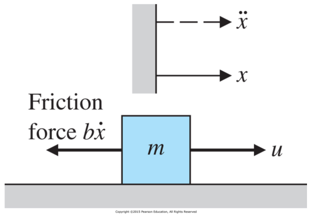
where the state vector is defined as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and A is called a system matrix, B an input matrix, C an output matrix, and D a direct transmission term.

• (Example 7.2) Consider Figs. 2.1 and 2.2, the automotive cruise model is obtained as





$$m\ddot{x} = u - b\dot{x} \qquad \rightarrow \qquad m\ddot{x} + b\dot{x} = u \qquad y = x$$

Let us assign states as follows:

$$x_1 = x \qquad\qquad x_2 = \dot{x}$$

Take the time derivatives

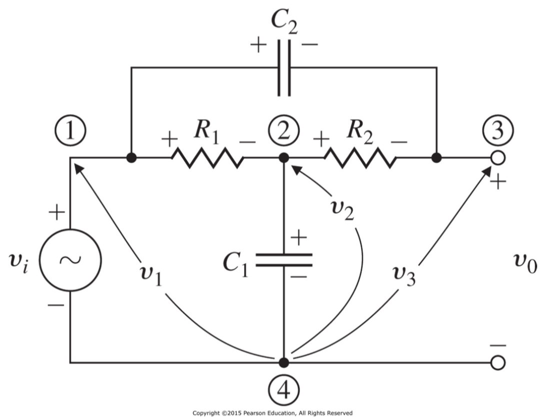
$$\dot{x}_1 = \dot{x} = x_2 \qquad\qquad \dot{x}_2 = \ddot{x} = -\frac{b}{m}\dot{x} + \frac{1}{m}u = -\frac{b}{m}x_2 + \frac{1}{m}u$$

Let us obtain state-variable equation as the vector equation:

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u \qquad\qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u$$

$$\dot{x} = Ax + Bu \qquad\qquad y = Cx + Du$$

- (Example) Consider Fig. 2.26 in the textbook, the bridged tee circuit model is obtained as



$$\frac{v_2 - v_i}{R_1} + \frac{v_2}{1/(C_1 s)} + \frac{v_2 - v_o}{R_2} = 0 \qquad \frac{v_o - v_i}{1/(C_2 s)} + \frac{v_o - v_2}{R_2} = 0$$

If we take differential equation form, then we have

$$\frac{v_2 - v_i}{R_1} + C_1\frac{dv_2}{dt} + \frac{v_2 - v_o}{R_2} = 0 \qquad C_2\frac{d(v_o - v_i)}{dt} + \frac{v_o - v_2}{R_2} = 0$$

also if we assume that $v_{C1} = v_2$ and $v_{C2} = v_i - v_o$, then the above can be rewritten as

$$\begin{aligned}
\frac{dv_{C1}}{dt} &= -\frac{v_2 - v_i}{C_1R_1} - \frac{v_2 - v_o}{C_1R_2} = -(\frac{1}{C_1R_1} + \frac{1}{C_1R_2})v_{C1} + \frac{v_i}{C_1R_1} + \frac{v_i - v_{C2}}{C_1R_2} \\
&= -(\frac{1}{C_1R_1} + \frac{1}{C_1R_2})v_{C1} - \frac{1}{C_1R_2}v_{C2} + (\frac{1}{C_1R_1} + \frac{1}{C_1R_2})v_i \\
\frac{dv_{C2}}{dt} &= \frac{v_o - v_2}{C_2R_2} = \frac{v_i - v_{C2} - v_{C1}}{C_2R_2} = -\frac{1}{C_2R_2}v_{C1} - \frac{1}{C_2R_2}v_{C2} + \frac{1}{C_2R_2}v_i
\end{aligned}$$

Let us assign states as follows:

$$x_1 = v_{C1} \qquad\qquad x_2 = v_{C2}$$

Let us obtain state-variable equation as the vector equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1R_1} + \frac{1}{C_1R_2} & -\frac{1}{C_1R_2} \\ -\frac{1}{C_2R_2} & -\frac{1}{C_2R_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1R_1} + \frac{1}{C_1R_2} \\ \frac{1}{C_2R_2} \end{bmatrix} u$$

$$\dot{x} = Ax + Bu$$

and the output equation is

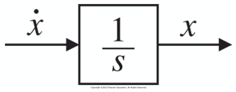
$$y = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1 \cdot u$$

$$y = Cx + Du$$

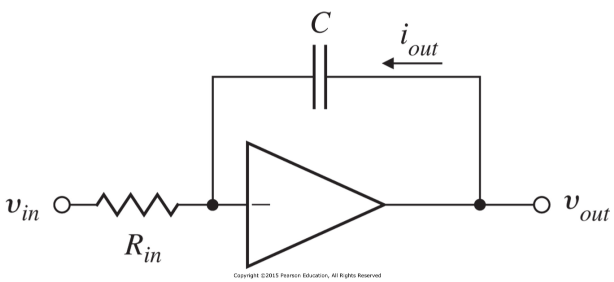
- (Example 7.3)
- (Example 7.4)
- (Example 7.5)
- (Example 7.6)

3 Block Diagrams and State-Space

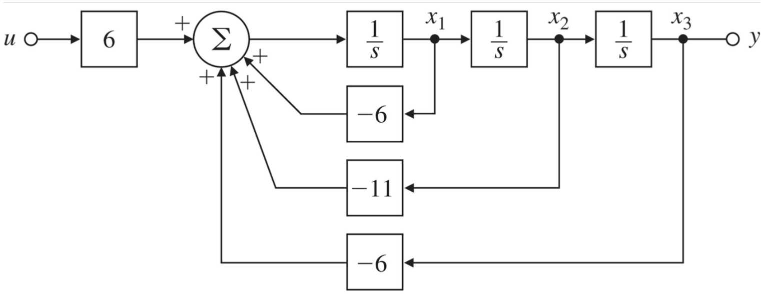
- See Fig. 7.3 for the integrator as the central element.



- Integrator is constructed from an OP-Amp with a capacitor feedback and a resistor feed-forward as shown in Fig. 2.29



- (Example) Find a state-variable description and the transfer function of the system shown in the following figure whose differential equation is



$$y^{(3)} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$

From the figure, we know that

$$\begin{aligned} y &= x_3 \\ \dot{x}_3 &= x_2 \\ \dot{x}_2 &= x_1 \\ \dot{x}_1 &= -6x_1 - 11x_2 - 6x_3 + 6u \end{aligned}$$

The state-space description becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} u \qquad y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0 \cdot u$$

The transfer function is

$$s^3Y(s) + 6s^2Y(s) + 11sY(s) + 6Y(s) = 6U(s) \qquad \rightarrow \qquad \frac{Y(s)}{U(s)} = \frac{6}{s^3 + 6s^2 + 11s + 6}$$

- (Example 7.7)